THE

PHYSICAL SOCIETY

OF

LONDON.

PROCEEDINGS.

VOLUME XXXIII.—PART II.

FEBRUARY 15, 1921.

Price to Non-Fellows, **6s.** *net, post free* **6/3**.

Annual Subscription, **30/-** *post free, payable in advance.*

Published Bi-Monthly from December to August.

LONDON:
FLEETWAY PRESS, LTD.,
3–9, DANE STREET, HIGH HOLBORN, W.C.1.

1921.

THE PHYSICAL SOCIETY OF LONDON.

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V. *Some Slide Rule Improvements.* *By* J. ST. VINCENT PLETTS.

RECEIVED SEPTEMBER 23, 1920.

ABSTRACT.

The relation existing between the log-log scale and the log scale in slide rules at present on the market is dealt with, and it is shown that if this relation is made that of the common logarithm the properties of characteristics and mantissæ enable the range of the non-recurring log-log scale to be indefinitely extended. In addition an equally divided scale is arranged to give ae^x and $\log_e ax$, where a is a number on the log scale, and scales for all the circular and hyperbolic functions are arranged on the back of the slide so as to read on the log scale. All such functions as $e^n \sin x$ and $\log n \cosh x$ are, therefore, obtained with a single setting of the slide and cursor. Other minor modifications are made, and a cursor for magnifying and subdividing the divisions is described.

THERE are now several slide rules on the market with log-log scales, by means of which any number can be raised to any power, or the logarithm of any number to any base can be obtained. As, however, the log log scale is non-recurring it is usually made two or three times the length of the rule, with the result that confusion sometimes arises as to which part of the scale is being used. The endeavour to get a suitable range of numbers on this scale has also led to curious relations between it and the log scale with which it corresponds. The numbers on the latter are, of course, the logarithms of the numbers opposite them on the former; but, taking three well-known examples, the base of these logarithms is 2 in the Jackson-Davis double slide rule, 2·5 in the Perry rule, and 3·5 in the Yokota slide rule. It is interesting to note that in the original Roget slide rule, as published in the "Philosophical Transactions" for 1815 (Part I., pages 9 to 28), the base was 10; but the scale was in two parts, running from 1·0025 to 1·26, and from 1·26 to 10^{10} respectively.

It occurred to me some time ago that the range of the log-log scale might be indefinitely extended, without making the scale longer than the rule, by taking advantage of the properties of characteristics and mantissæ of common logarithms. To this end I made a slide rule (Fig. 1), in which the relation between these scales was that of the common logarithm and

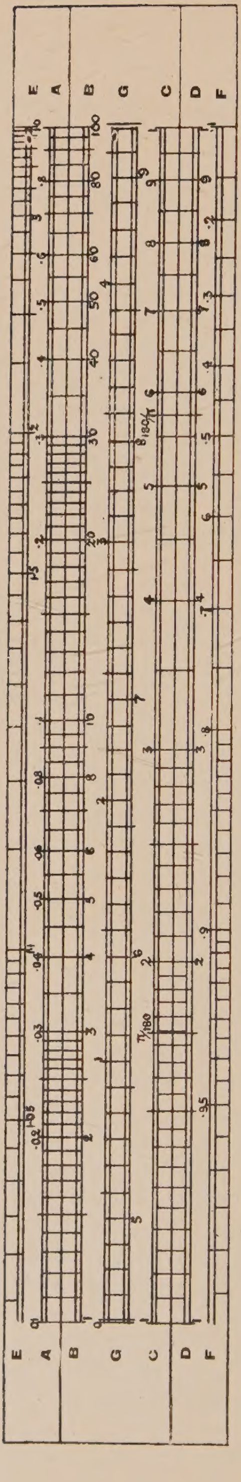
E
A
B
G
C
D
F
π/180
180/π

Fig 1.

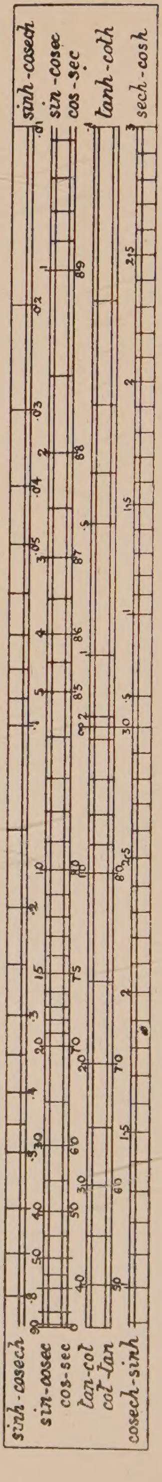
sinh-cosech
sin-cosec
cos-sec
tan-cot
cot-tan
cosech-sinh
sinh-cosech
sin-cosec
cos-sec
tanh-coth
sech-cosh

Fig. 2.

the highest number on the log-log scale (E) was 10. It will be seen that common logarithms can be read directly from these scales ; all that is necessary, if the number is greater than 10, being to add the appropriate characteristic. If the number is less than unity the characteristic will be negative, or the true negative logarithm can be obtained from the corresponding reciprocal scale (F). If it be desired to raise any number to any power, one proceeds in the ordinary way ; but if the result is greater than 10 it cannot be read on the log-log scale. All that is necessary, however, is to use the other end of the slide, when the logarithm of the result can be read on the log scale (A), and by dropping the first figure or characteristic of this logarithm the result can be read on the log-log scale opposite the remaining mantissa on the log scale, it being, of course, necessary to move the decimal point in accordance with the characteristic dropped. Thus, the range of the log-log scale is indefinitely extended in the upward direction, and of the reciprocal scale in the downward direction, by the exercise of no greater mental effort than is required to remember the position of the decimal point in ordinary calculations with the slide rule.

I have found it advantageous to put in the decimal points of the numbers on the upper log scales (A & B). This does not interfere with ordinary multiplication or division, which is nearly always performed on the lower log scale (C & D), and, besides making the common logarithms direct reading, it has several other advantages which will appear later. One of them can indeed be seen now, as there is no longer any doubt as to which end of the scale should be used for the extraction of square roots.

Having got the log-log scales into the length of the rule in a single part without limitations, I found that, though functions of the form e^{ax} were easily dealt with, those of the form ae^x, which are, perhaps, of more frequent occurrence, entailed more labour, and were more liable to error because of the necessity of transferring an intermediate result from one scale to another. I accordingly placed along the middle of the slide an equally divided scale (G) the numbers on which were the natural logarithms of the numbers opposite them on the log scale, so that e^x and log x were direct reading, and ae and log ax could be obtained with a single setting of the slide. It will be seen that by relating this equally divided scale to the upper log scale (B) on the slide, on which the position of the decimal point has already been indicated, a very useful scale is obtained, which can be made twice its apparent length by numbering on both sides, since $e^{4\cdot6}$ is very nearly equal to 100.

Now since we can either multiply by e^x, or take the natural logarithm of any number on the upper log scale, it follows that any function reading on this scale can be similarly treated. I have therefore rearranged the ordinary sine and tangent scales (Fig. 2), which are as usual placed on the back of the slide, and have added other scales for the hyperbolic functions so that they all read on the upper log scale in front.

The scales for the circular functions might have been divided into radians or into degrees and minutes as is usual, but as conversion from one into the other by means of the slide rule gives and requires degrees and decimals of a degree, I have found it most convenient to divide the scales in this way, while I have also marked each tenth of a radian on these scales, so that conversion is often not necessary at all.

The arrangement of all these scales on the back of the slide is such that the values of the functions are in every case read on the fixed portion (A) of the upper log scale in front if they are less than unity, and on the sliding portion (B) of that scale if they are greater than unity, so that the position of the decimal point is always given. From this and from what has already been said about the equally divided scale (G) along the middle of the front of the slide it will be seen that the values of all such functions as $e^x \sin x$ and $\log a \cosh x$ can be read direct from the rule, and generally with a single setting of the slide and cursor, so that by these means the power of the slide rule is considerably extended, and it becomes a much more valuable mathematical instrument.

These scales on a 25 cms. rule give sufficient accuracy for all ordinary purposes, and it is hardly practicable to increase the length of a straight rule sufficiently to get another decimal place in the results, but several attempts have been made to increase the power of subdividing the scales. There is the well-known magnifying cursor, which enables the subdivision to be more accurately estimated by the eye, and there is also the Goulding Cursor, by means of which the subdivision can actually be measured, though I have not found it more accurate than the unaided eye. The disadvantage of the ordinary magnifying cursor is that the semi-cylindrical lens gives such distortion near its edges that it is difficult to tell exactly which part of the scale is being magnified unless one of the figures upon the scale is also in view. To obviate this disadvantage I have made a lens which is as wide as an ordinary cursor, and the upper surface of which is curved so as to give sufficient magnification in the middle and so little distortion towards the edges, that everything on the scale within the width is easily seen.

As such a lens is raised off the scale to a distance within its focal length the magnification increases, and I have made use of this fact for the purpose of subdividing the scale. On the under side of the lens I place a finely subdivided division, which can be fitted to any division of the scale by simply raising the lens until the apparent size of the two is the same, so that another decimal place is easily and accurately obtained.

But I have found it impossible to get a really satisfactory lens manufactured, and the compromises I have had made are altogether too expensive to form part of the cursor of a commercial slide rule.

DISCUSSION.

Mr. F. J. W. WHIPPLE expressed his appreciation of the Paper. He pointed out to the author that there was a slide rule issued by the Meteorological Office in which the degrees were divided decimally. Could a little more information be given of the significance of the figures. The Paper, as it stood, was a little difficult to follow.

Dr. VINCENT suggested a modification of the cross-section of the rule so that four of the scales could be readily marked off on paper for graphical purposes. This was the principal use that many workers had for these scales, and on the ordinary rule none were easily transferable.

Dr. ECCLES asked how the accuracy of the new log log scale compared with the ordinary folded one.

The AUTHOR said he was interested to hear of the Meteorological Office rule. Dr. Vincent's suggestion could probably be better carried out on a special rule for the purpose he mentioned.

VI.—*The Current Density in the Crater of the Carbon Arc.*

By N. A. Allen, *B.Sc.*

Received May 27, 1920.

(Communicated by D. Owen, B.A., D.Sc.)

Abstract.

In this work the Y-arrangement of carbons due to Forrest was used. This facilitated accurate measurement of crater area. The carbons used were solid special electra of diameter 6 mm. For currents up to 10 amperes the crater area was directly proportional to total current, thus maintaining a constant current density. This was 0·746 ampere per square millimetre.

The object of the present research is an attempt to eliminate the uncertainty attending an investigation of the crater area, and the current density in the crater, of the carbon arc.

Mrs. Ayrton* found that a straight line law approximately connects crater area and current, but did not form any equation expressing the law, on account of possible errors in her measurements.

Current in amperes.			4	7	10	15	20
Projected arc lengths in mm.	1	Ayrton	7·56	13·85	14·2	23·3	32·2
		Rasch & Tornberg	7·74	12·25	16·55	23·76	30·97
	2	Ayrton	11·35	13·85	17·7	24·6	32·2
		R. & T.............	9·19	13·66	17·95	25·25	32·47
	3	Ayrton	9·89	13·85	—	22·5	32·2
		R. & T.............	9·9	14·39	18·7	25·88	33·08
	4	Ayrton	9·89	16·65	18·9	26·4	34·2
		R. & T.	10·35	14·86	19·17	26·42	33·59

Rasch and Tornberg† obtained similar results to those of Mrs. Ayrton, and the two series of measurements are tabulated below, areas in square millimetres.

Mrs. Ayrton used cored positives, Rasch cored negatives.

The lines representing these figures approximately, are shown in Fig. 3, for comparison with the results of the present work, which was done with arc lengths of 1 to 10 mm., the arc being always perfectly silent. Solid carbons were used.

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* "The Electric Arc," p. 151, *et seq.*

† "Electric Arc Phenomena," p. 143, *et seq.*

The chief difficulties met in a measurement of crater area are as follows :—

1. The crater cannot be viewed directly.
2. The area must be calculated from an oblique measurement of crater diameter, assuming the crater to be truly circular, which is by no means certain.
3. The edges of the crater may not be well defined.
4. The crater is curved inwards, so that the true area is greater than that observed.
5. The presence of fumes and carbon vapour in the crater may cause error due to refraction.

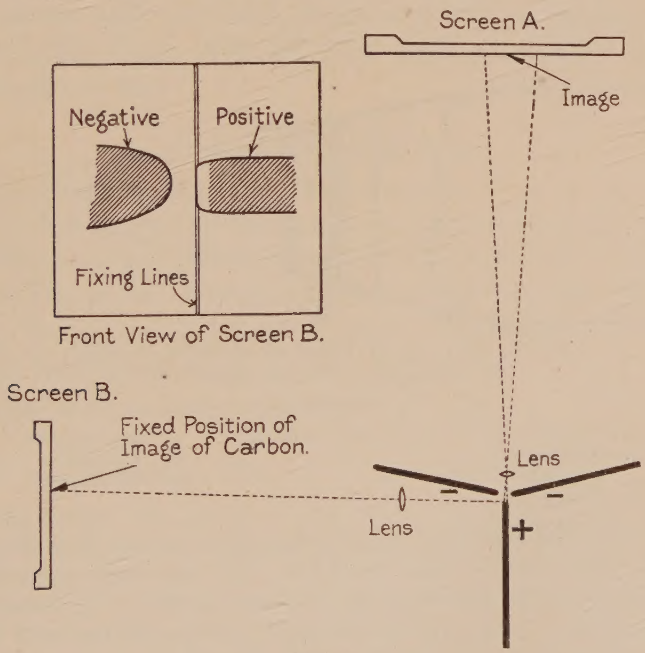

Fig. 1

The last two difficulties disappear if apparent crater area is required, as in photometric work. None of these causes of inaccurate results, however, are found with the Y-arrangement of carbons originated by Forrest,* and used in the present work.

Messrs. Paterson, Walsh, Taylor and Barnett† assumed that the positive crater covers the whole of the end of the carbon for all currents when the current density in the actual carbon exceeds 0·2 amperes per square millimetre. Figures since communicated, however, show that for currents of 100 to 220 amperes the true density in the crater was sensibly constant, which agrees with the results of the present work, which was carried out with much smaller currents, the true density in their case being about 0·45 amp. per sq. mm.

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* " Electrician," 1913, Vol. LXXI., pp. 729 and 1007.

† " Proc." Inst. El. Eng., Vol. LVIII.

Methods Adopted.

The arc was always maintained absolutely silent, and before readings were taken, 10 or 15 minutes was always allowed for the carbons to burn to a steady condition. The current in each negative was kept about the same, but it was found that the positive crater was unchanged when one took 50 per cent. more current than the other. The arc length was that which gave perfect silence for any one current. An image of the positive crater was thrown on a screen, and the area was measured by making many points round the circumference with a fine pencil.

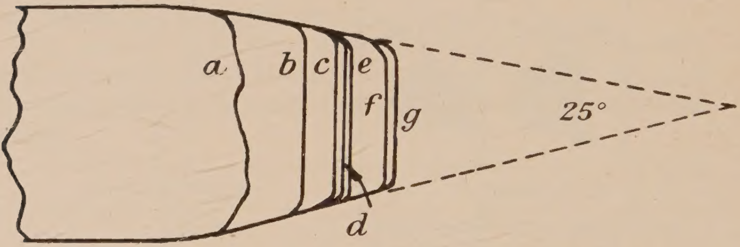

Fig. 2.—Tracing of Image of Positive Carbon.
(Magnification = 5·2 times.)
Crater Face.

a. 12 ampere (unsteady, hissing violently).
b. 9·0 ampere.
c. 7·0 ampere.
d. 6·5 ampere.
e. 6·0 ampere.
f. 4·8 ampere.
g. 4·5 ampere.

Since with the apparatus in use, an error of 1 mm. backward or forward in the position of the positive crater would cause an 8 per cent. error in the area, the arrangement shown in Fig. 1 was devised. A screen B was placed at the side of the arc, and an enlarged image of the positive carbon focussed upon it. Two fine lines were drawn on this screen parallel and vertical, and the end of the image must just show in between for the carbon to be correctly disposed.

The magnification of the image on screen A was measured by placing a series of cylindrical gauges having flat polished ends, with their faces in turn in the correct position, and measuring the image of the gauge when brilliantly illuminated.

Calibration of Magnification.

Gauge.	Diameter in mm.	Magnification.
1	4·782	31·72
2	3·210	31·49
3	2·739	31·73
4	1·680	31·56

Distortion being thus negligible, the mean magnification of 31·62 was used.

Limits of Accuracy.

Possibility of inaccuracy was as follows :—

(*a*) Due to error in position of carbon, less than 0·1 mm. causing area to be subject to an error of 0·75 per cent.

(*b*) Due to error in magnification ; less than 0·1 per cent.

(*c*) Due to error in measuring image —0·6 per cent.

(*d*) Error in observing current —0·25 per cent.

Total error probably well within 1 per cent.

Results (*a*) *Shape of Carbon and Crater.*

The crater was always a true circle. It is suggested that this is due to the mutual attraction of parallel streams of current flowing in the same direction. It was found impossible to maintain two separate arcs on the end of the same positive.

A tracing was made at several currents of the projection of the positive carbon. This showed that in all cases when the arc has burned to a steady condition, the carbon tapers down uniformly at an angle of about 25 deg. (*see* Fig. 2), the crater occupying the whole of the reduced end of the carbon. The crater was flat and quite smooth, and perpendicular to the axis of the carbon, as evidenced :—

1. By the appearance of the carbon when examined, and
2. By the fact of the whole crater being in focus on the screen when projected.

Results (b) Crater Area and Current Density.

On Fig. 3 are plotted all the observations made, and for comparison results by previous investigators are given also.

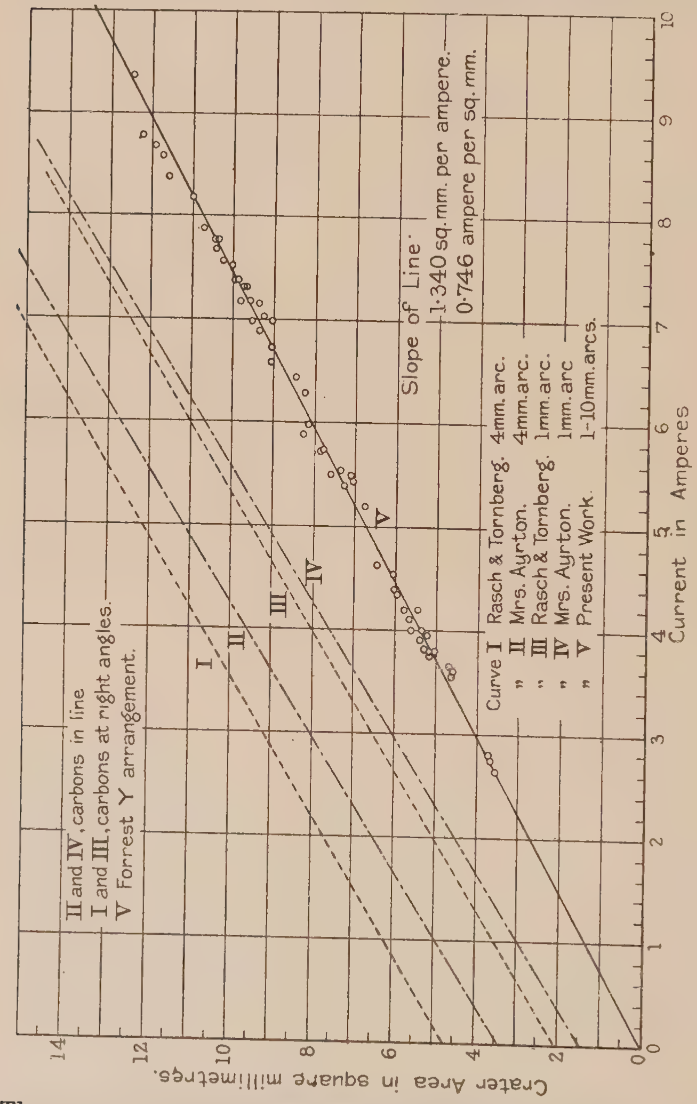

FIG. 3.

The current densities have been worked out for Fig. 4. The maximum deviation from the mean on any observation in a

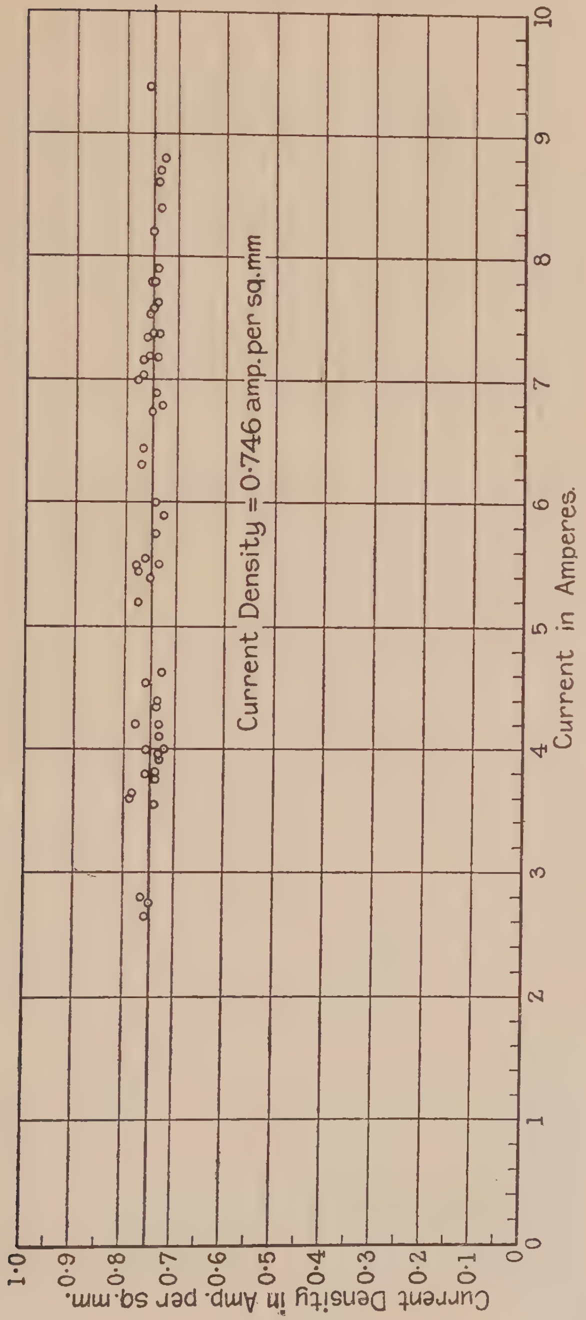

FIG. 4.—THE CURRENT DENSITY IN THE CRATER OF THE CARBON ARC.

series of 60 readings of current density was only 5 per cent., and 70 per cent. of all results were within 2 per cent.

The carbons used were solid special electra carbons marked [– – – – – – – – – – ELECTRA] of diameter 6 mm., and as a conclusion to the present research it may be stated that for currents up to 10 amperes, using the Y-arrangement of carbons :

(*a*) Crater area is directly proportional to total current, and

(*b*) The current density in the positive crater is a constant and, correct to 1 per cent., is equal to 0·746 amperes per square millimetre.

If A=crater area in square millimetres,
I=current in amperes,
then $A=1{\cdot}340\,I$.

This research was carried out in the Electrical Engineering Laboratory at the East London College, and the best thanks of the author are due to Prof. J. T. MacGregor-Morris for suggesting this line of investigation, and for some very helpful criticisms and suggestions during the work.

DISCUSSION.

Prof. MORRIS read a portion of a letter from Mrs. Ayrton, who considered the Paper of great importance, and that the results were what she would have expected from accurate determinations.

The speaker mentioned that the carbons used were very pure. The effect of quality of carbon on current density had to be investigated.

The late Mr. Forrest, of whose work this was a continuation, showed that the candle-power per square millimetre was 172 to 174. Combining this result with that of the present Paper, we find that the candle-power per ampere is 232. He suggested, therefore, that the Y-type of arc gives a standard of light, requiring only an ammeter and good quality carbons, constant to at least 1 per cent.

Prof. RANKINE asked if the result was unaffected by the mass of carbon carrying off the heat.

Prof. MORRIS said the result was certainly true from 6 to 10 amperes.

Dr. BRYAN asked how the result was affected by the presence of areas of special brightness or darkness. He had frequently noticed such areas.

Mr. F. J. W. WHIPPLE asked if Prof. Morris could give an idea of the physics of the constancy of current density.

Dr. D. OWEN said he was going to refer to the same question. The crater was stated to be perfectly circular, and the edge was very sharp, judging from the photograph shown on the screen. This was surprising. One would have expected some sort of penumbral region. A comparison of the ionic conditions in the arc with those in a conducting liquid was of interest. $n\,e\,u$=constant where n is the number of charged particles per unit volume, e the charge on each, and u their velocity. In the arc, as the current increases the voltage diminishes, and therefore the velocity u decreases. Consequently n must increase. In the case of electrolytic conduction n is constant.

Mr. J. W. T. Walsh (communicated): I much regret that another engagement prevents my being present when Mr. Allen's very interesting and valuable Paper is read.

The results obtained are most surprising in their regularity and the accuracy with which the crater area remains proportional to the current for all the arc lengths employed. The current density found is much higher than that generally recognised hitherto as the limits of steady burning for arcs of large currents, say, from 100 to 200 amperes. It would be most valuable if the author could see his way to extending his researches to include currents of such magnitude as this. The work done at the National Physical Laboratory was practically confined to large current arcs, and the difficulties experienced in obtaining steady burning conditions with these show that the smaller current arcs behave much more regularly, and are, therefore, probably capable of smooth running at much higher current densities than the larger arcs. In this connection I should like to express my thanks to Prof. Morris, who very kindly gave me an opportunity of seeing Mr. Allen's apparatus working at East London College. The smoothness of running of the arc, and the uniformity of brightness of the crater were remarkable and I feel sure that everyone who has worked on the carbon arc will be more than interested in the results presented this evening, and will look forward to further work by the author on similar lines.

The Author communicated the following: In reply to Prof. Rankine, the carbons used were all the same size—6 mm. diameter.

The effects mentioned by Dr. Bryan, which are due to inequalities in the carbons, do not affect the results, as when a bright or dark area is present the arc is unsteady, and one has been careful to avoid taking readings under unstable conditions.

Dr. Owen assumed that as the current increases the voltage diminishes. This is true when the length of the arc is kept constant, but in the case of these silent arcs, length is approximately proportional to current, hence as the current increases, the voltage remains fairly constant. Then in the equation used by Dr. Owen ($n\,e\,u =$ const.), since the voltage is constant, therefore the velocity is constant, hence the number of ions or charged particles is constant.

Another interesting result following from the relation between current and length for silence is as follows: Mrs. Ayrton (" Electric Arc," p. 158) suggested that the equation connecting arc current and area was

$$A=\alpha L+\beta I. \qquad (A=\text{area},\ L=\text{length},\ I=\text{current.})$$

If $L=\gamma I$,

then $A=\text{constant}\times I$, as the present work shows.

Mr. Walsh mentions that the current density obtained was higher than he would have expected. This high current density appears to be due largely to the reactions of the arc streams upon each other, forcing up the current density to a high value.

VII. *A Sodium Vapour Electric Discharge Tube.*

By F. H. Newman, *M.Sc., A.R.C.Sc., University College, Exeter.*

Received September 6, 1920.

Abstract.

A simple method of producing a bright source of sodium radiation is described. An electric discharge tube of quartz, having a pool of sodium-potassium alloy as one electrode, and an iron rod as the other electrode, is utilised. The sodium vapour is proeduced by gently heating the lower part of the tube, containing the alloy, over a ring gas burner concentric with the tube. When an electric discharge is passed, a bright yellow light is emitted. This light consists almost entirely of the *D* lines. Any gases present in the tube are absorbed by the alloy while the discharge is passing. No continuous pumping is necessary to keep the tube exhausted, and the experiment works best when the temperature of the sodium vapour is 250°C.

1. Introduction.

In order to stimulate the resonance radiation in a bulb of sodium vapour, the exciting light must consist of very narrow bright *D* lines. A Bunsen flame to which salt has been added is not a suitable source, since the centre of the line is dark. The lines themselves are very broad, and the exciting radiation is not confined to the actual region of the spectrum required. When a sodium arc in vacuum is substituted for the flame, this difficulty is overcome. Lord Rayleigh * has constructed and exhibited a sodium vacuum arc in quartz, which is similar in principle to the mercury lamp. The sodium lamp is more troublesome to construct and work than the mercury lamp, and requires continuous pumping to keep it exhausted while the lamp is running. Iron electrodes are unsuitable, since they fuse and drop off after the lamp has been in use for an hour or two. Tungsten, which seems to withstand the action of sodium vapour, is used instead of iron.

The apparatus described below works well, and although it does not give as brilliant a source of light as the sodium arc, yet it can be quickly constructed, and requires no continuous pumping while the electric discharge is passing.

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* Hon. R. J. Strutt, "Proc." Roy. Soc., A, Vol. XCVI. (1919).

2. APPARATUS.

The form of discharge tube used is shown in Fig. 1, and the electric discharge through sodium vapour is used. The tube is of quartz ; one of the electrodes (*B*) is a pool of the sodium-potassium alloy ; the other electrode is of iron, and an iron wire is used to make electrical connection with the alloy. The latter is introduced into the tube through a side tube (*D*). Throughout an experiment (*D*) is closed by means of a quartz plate. When an electric discharge is passed, the scum which is formed at the surface of the alloy by oxidation, disappears, and a clean surface is obtained. The alloy absorbs all gases in the tube except argon and helium, when the pressure is low, and any gas liberated, while the electric discharge is passing, is at once absorbed by the alloy. As a result the pressure of the gas in the tube is always very low. To obtain the sodium vapour the alloy must be heated to 250°C. to 300°C., as the heat generated by the electric discharge is not sufficient to produce the vapour. A small gas ring is suitable as a source of heat. When a condensed discharge passes through the sodium vapour, a brilliant yellow light is produced. The light is intensified by making a constriction in the tube at (*C*). When examined with a

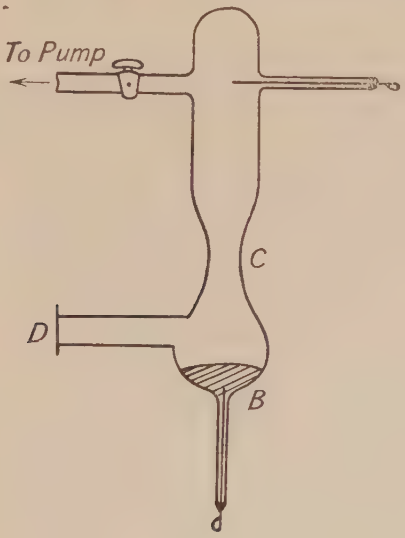

FIG. 1.

spectroscope, the potassium spectrum is very faint, almost all the light coming from the *D* lines of sodium. The faintness of the potassium spectrum is due to the vapour pressure of potassium being less than that of sodium at the temperature of the experiment.

While the discharge tube is running, the sodium and potassium vapours condense on the colder parts of the tube. This is not a serious disadvantage, and the condensation can be eliminated by enclosing the tube within an electrical heater. The iron electrode is not affected by the sodium vapour, even after the tube has been in use for several hours. No continuous pumping is required while the discharge is passing, as, after the initial exhaustion of the tube, the action of the alloy keeps the vacuum high. This is a great advantage, as hydrogen is always freely emitted from the sodium-potassium alloy, when the latter is heated. With the temperatures stated above there is no reduction of the silica. In practice, the alloy should be made the cathode, as it absorbs gas at a greater rate when made the cathode than when it is the anode. The light cannot be viewed through the window at (*D*), as the vapour condenses on it after a short time.

Using this discharge tube, the light from it was focussed on a glass bulb containing sodium vapour. To vary the density of the vapour, the glass bulb was enclosed in an electric heater. At a temperature of 250°C., brilliant fluorescence of the sodium vapour was stimulated. The fluorescence was photographed (Fig. 2). The light patches shown are due to reflected lights. As the temperature of the glass bulb was raised, the fluorescence across the tube decreased, owing to the increased density of the vapour. Heating the side of the bulb with a Bunsen flame the fluorescence moved away from the locality heated. The discharge tube is a suitable source of sodium radiation for interferometer experiments.

DISCUSSION.

Dr. Bryan commented on the quality of the silica tap in the author's apparatus, which had evidently remained at a high vacuum for a considerable time.

Mr. Newman said the tap had probably been manufactured by luck, as some previous ones he had had were very bad. The grease he used was made of bees wax and vaseline.

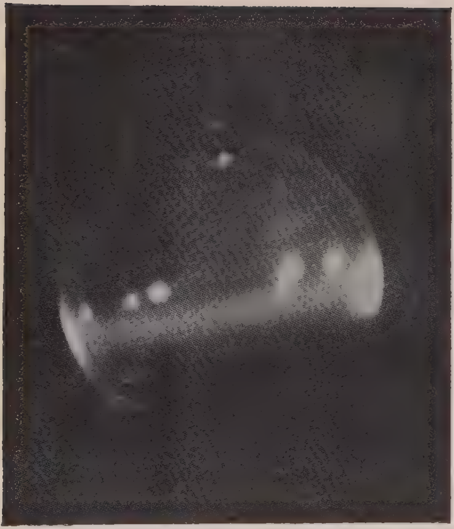

Fig. 2.—Fluorescence of Sodium Vapour. Exciting Light from the Sodium Vapour Electric Discharge Tube.

[*To face* p. 72.

VIII. *Absorption of Gases in the Electric Discharge Tube.*

By F. H. NEWMAN, *M.Sc., A.R.C.Sc., University College, Exeter.*

RECEIVED JULY 20, 1920.

ABSTRACT.

Various elements were deposited in a pure condition on the cathode in an electric discharge tube. To obtain a clean surface of the element, the latter was distilled in vacuum. An electric discharge was passed through nitrogen gas contained in the tube. Measurements were made to compare the amount of gas absorbed by the element in the tube with the quantity of electricity passing in the circuit. Potassium, sodium, mercury, cadmium, antimony, magnesium, calcium, zinc, tin, phosphorus, sulphur and iodine were tested in this way. The rates of absorption were very great with the last three elements. Hydrogen gas was also used in the tube, and absorption occurred with phosphorus, sulphur and iodine. The absorption of the gases is due principally to chemical action, and is attributed to the nitrogen and hydrogen assuming active modifications on the passage of an electric discharge, the gases being at low pressure.

1. INTRODUCTION.

THE present investigation is a development of the experiments described in "Proc." Roy. Soc., A., Vol. XC. (1914), and Proc. Phys. Soc., Vol. XXXII. (1920). In these Papers it has been shown that sodium, potassium and the liquid alloy of sodium and potassium absorb nitrogen when these metals are used as electrodes in the electric discharge tube. The gas is absorbed at a greater rate if the metal is used as the cathode. The quantity of gas absorbed, per unit quantity of electricity passing through the discharge tube, was measured at various pressures of the gas. No absorption occurred unless the surfaces of the metals were quite clean. Sodium and potassium were distilled in vacuum in order to obtain surfaces which were not contaminated. Other elements, in the molten condition, had been tested without success; this was traced to the contamination of the surfaces, and these elements are re-examined in the present work. The same apparatus, as described in the above Papers, was used, but the form of the discharge tube was changed, and is shown in Fig. 1. The electrodes *A* and *B* were of aluminium. To the end of the cathode *B* was attached a small disc of platinum foil. The element under examination was placed at the bottom of the tube, and by distilling it in vacuum, a thin layer of the element was formed on the foil.

2. Experiments.

A clean surface of the element having been formed on the cathode, a known quantity of nitrogen was admitted into the discharge tube. The method of measuring this amount of gas has been described in the previous Papers. The electric discharge was then passed through the gas, and the amount of hydrogen liberated in the voltameter gave a measure of the quantity of electricity passing in the circuit. The absorption of gas during the passage of the discharge was noted. In some cases it was necessary to maintain the discharge tube at a low temperature, by placing it in an enclosure surrounded with a freezing mixture. The nitrogen used in the experiments was prepared by allowing air to stand over phosphorus for two or three days. The hydrogen was obtained by electrolysis. Both gases were examined at various pressures, and the results obtained are shown in Tables I. and II.

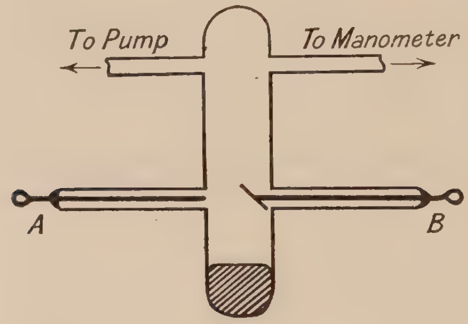

FIG. 1

3. Absorption of Nitrogen.

Lead, thallium and arsenic did not absorb nitrogen. The elements were tested with the gas at different pressures and temperatures.

TABLE I.—*Absorption of Nitrogen.*

Element.	Pressure in mms. Hg.	Temperature.	Volume of N_2 absorbed in c.c. (A)	Volume of H_2 liberated in Voltameter in c.c. (B)	Ratio $\frac{A}{B}$	Pressures at which absorption takes place in mm.s Hg.
Mercury	1·3	−13°C.	0·40	2·21	0·2	0·5—1·5
	0·9		0·36	0·92	0·4	
	0·6		0·33	0·51	0·6	
Cadmium	1·1	30°C.	0·18	0·13	1·4	1·1—1·8
	1·8		0·29	0·21	1·4	
Arsenic Lead Thallium	No absorption					
Antimony	1·8	−15°C.	0·65	0·45	1·4	1·2—1·9
	1·5		0·31	0·22	1·4	
Magnesium	3·4	50°C.	0·07	0·11	0·6	1·0—4·3
	2·0		0·17	0·19	0·9	
	1·9		0·31	0·16	1·9	
	1·3		9·68	0·24	2·8	
Calcium	1·8	−12°C.	0·14	0·12	1·2	1·1—1·9
	1·2		0·19	0·14	1·3	
Zinc	1·7	−12°C.	0·13	0·10	1·3	1·2—1·8
	1·5		0·20	0·14	1·4	
Tin	1·7	0°C.	0·28	0·57	0·5	0·1—2·0
	1·2		0·14	0·23	0·6	
	0·9		0·17	0·17	1·0	
Phosphorus......	13·3	−12°C.	0·37	0·03	12·3	X-Ray stage to 20 mm.s
	10·1		0·27	0·03	9·0	
	7·3		0·29	0·04	7·2	
	4·0		0·14	0·05	2·8	
	2·5		0·15	0·07	2·1	
	1·0		0·27	0·13	2·1	
Sulphur	11·6	−12°C.	0·23	0·04	5·8	0·1—12·0
	7·9		0·15	0·04	3·8	
	3·2		0·13	0·04	3·2	
	1·6		0·08	0·03	2·7	
Iodine	16·4	−20°C.	0·44	0·08	5·5	1·0—18·0
	12·2		0·29	0·07	4·1	
	8·3		0·27	0·08	3·4	
	4·1		0·31	0·11	2·8	
	1·9		0·23	0·11	2·1	
Bismuth	1·7	−14°C.	0·43	0·36	1·2	1·1—1·8
	1·4		0·22	0·17	1·3	

TABLE II.—*Absorption of Hydrogen.*

Element.	Pressure in mm.s Hg.	Temperature.	Volume of H_2 absorbed in c.c. (A)	Volume of H_2 liberated in voltameter in c.c. (B)	Ratio $\frac{A}{B}$	Pressures at which absorption takes place in mm.s Hg.
Phosphorus ...	12·8	−12°C.	0·28	0·03	9·4	X-ray stage to 18·0 mm.s
	11·2		0·25	0·03	8·3	
	8·1		0·29	0·05	5·8	
	3·7		0·26	0·09	2·9	
	2·4		0·23	0·11	2·1	
Sulphur	12·1	−12°C.	0·37	0·05	7·4	0·5—12·0
	8·3		0·13	0·03	4·3	
	3·1		0·37	0·11	3·4	
	1·4		0·29	0·12	2·4	
Iodine	14·6	−20°C.	0·39	0·09	4·3	1·0—15·0
	11·8		0·40	0·12	3·3	
	7·3		0·23	0·07	3·3	
	3·1		0·25	0·09	2·8	
	2·3		0·27	0·11	2·5	

Mercury absorbed nitrogen when the pressure of the gas was between 0·5 mm. and 1·5 mm. of mercury, if the tube was kept at —13°C. The amount of gas which could be absorbed at a time was very small. When the discharge was stopped, and the tube heated to about 20°C., all the gas which had disappeared was re-liberated. After the surface of the mercury had been exposed to the atmosphere, no absorption occurred, which showed the importance of having a clean surface of the element. Stark* has noted that mercury, used as an electrode in the discharge tube, absorbs nitrogen and oxygen.

Cadmium absorbed the gas at ordinary temperatures, providing the pressure was between 1·1 mm. and 1·8 mm. of mercury. Most of it was re-liberated on heating the tube. The effect in this case may be due to occlusion of the gas within the metal. On the other hand, there may be a chemical compound of cadmium and nitrogen produced, which compound dissociates easily on the application of heat. The total amount of gas which was absorbed at a time was very small.

Antimony absorbed the gas at low temperatures if the pressure varied from 1·2 mm. to 1·9 mm. of mercury. The amount of gas which disappeared before it was necessary to prepare a fresh surface was only 0·2 c.c., and the gas absorbed was evolved when the tube was heated to 50°C. Antimony resembles cadmium in this respect.

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* "Phys. Zeit.," Vol. XIV., p. 417 (1913).

When magnesium is heated to a red heat it combines with nitrogen, forming a nitride of magnesium. This fact was utilised by Rayleigh* in the separation of nitrogen and argon. At ordinary temperatures there is no such combination. When magnesium was used as the electrode in the discharge tube, nitrogen was absorbed if the pressure of the gas was between 1·0 mm. and 4·3 mm. of mercury. At low pressures the rate of absorption was very high. This absorbed gas was not re-liberated when the tube was heated to 300°C. After about 4 c.c.s of the gas had disappeared, the contents of the tube were mixed with caustic soda, and boiled. Ammonia gas was evolved, showing that magnesium nitride had been formed in the tube. The absorption in this case is due to chemical action. It was possible to absorb about 4 c.c.s of gas at atmospheric pressure before the action ceased.

Soddy† has shown that when calcium is first heated in vacuo, gaseous compounds of carbon, hydrogen and oxygen are given off ; if the heating be continued, absorption of the gases begins, accompanied by volatilisation of the calcium. Experiments were made by him on the power of calcium, to absorb different gases, and he found that all gases, except the argon group, were rapidly absorbed. In the discharge tube I found that nitrogen was absorbed at 12°C. if the pressure of the gas was between 1·1 mm. and 1·9 mm. of mercury. It was possible to absorb only 0·1 c.c. of gas at a time, but this small amount was due to the very thin layer of calcium deposited on the cathode. On heating the tube to 300°C. none of the gas was evolved. This suggests that chemical action accounts for the absorption in this case.

Zinc absorbed nitrogen at —12°C., providing the pressure of the gas varied from 1·2 mm. to 1·8 mm. of mercury. The total amount of gas which disappeared was very small, and most of it was re-liberated when the tube was heated.

Tin does not combine with nitrogen when it is heated in that gas, but the gas was freely absorbed by the metal in the discharge tube if the pressure was within the range 2·0 mm. —0·1 mm. of mercury. The rate of absorption was uniform, and was large compared with that obtained with other metals.

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* " Phil. Trans.," 186 (1895).

† " Proc." Roy. Soc., Ser. A., 78, p. 429 (1907).

It was possible to absorb 0·3 c.c. of gas at a time. When the tube was heated to 300°C. no gas was evolved from the metal, and the presence of a nitride of tin was found by the method described in the case of magnesium. This absorption of nitrogen by tin has not been observed previously.

Strutt* has shown that phosphorous absorbs nitrogen and hydrogen when an electric discharge is passed through a bulb containing this element. The gases were not re-liberated on heating, and he attributed the absorption to the formation of chemical compounds. The rate of absorption of these gases, compared with the quantity of electricity passing through the tube, is very great ; and if phosphorus is placed in a discharge bulb the vacuum is kept very high. In the present experiments the discharge tube was maintained at —12°C., in order to minimise the production of phosphorus vapour. The actual rate of absorption was about 4 c.c.s (at atmospheric pressure) in 15 minutes ; this rate decreased as the discharge continued, and the amount of gas which could be absorbed depended on the thickness of the phosphorus layer used in the experiment. As shown by Strutt, the gas was not re-liberated from the phosphorus when the tube was heated, and it appears that a chemical compound of nitrogen and phosphorus is formed.

With sulphur, a large rate of absorption was observed, and the absorption occurred over a range of pressures 12·0 mm.—0·1 mm. of mercury. By means of the discharge the pressure of the gas in the tube was lowered rapidly, and reached the X-Ray stage, provided a clean surface of sulphur was used on the cathode. When the sulphur was exposed to the atmosphere, the rate of absorption of the gas was less, and it was not possible to obtain such a final low-pressure in the tube. No re-liberation of the gas occurred when the contents of the tube were heated to 100°C. Of course the sulphur vaporised, but on allowing the contents to cool, the original volume was attained. This showed that chemical combination had occurred between the gas and sulphur. The absorption of the gas was greater if the film of sulphur on the cathode was formed by condensation of the vapour, than if the layer was produced by allowing melted sulphur to flow over the cathode.

The results obtained with iodine resembled those noted in the case of phosphorus. No absorption occurred if the pressure of the gas was below 1 mm. of mercury. On account of the sublimation of iodine, the tube was maintained at —20°C. After the surface of the substance had been exposed to the atmosphere, the rate of absorption decreased. There was no re-liberation of the gas when the contents of the tube were heated to 100°C., which indicated that the phenomenon was due to chemical action, and not to mere occlusion of the gas within the iodine.

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* "Proc." Roy. Soc., Ser. A., 87 (1912).

The results obtained with bismuth were similar to those found with antimony. The discharge tube was maintained at —14°C., and absorption occurred when the pressure was between 1·8 mm. and 1·1 mm. of mercury. Only 0·18 c.c. of gas was absorbed by any one surface, and all of the gas absorbed was evolved when the tube and its contents were heated to 40°C. The effect in this case appears to be due to occlusion.

4. Absorption of Hydrogen.

As shown in Table II. the only elements which absorbed hydrogen were phosphorus, sulphur and iodine. The amount of hydrogen which could be absorbed at a time by phosphorus, and the rate of absorption, both increased when the tube was warmed. In this case it seems that the chemical action between the solid and the gas is increased as the temperature rises. It is known that phosphoretted hydrogen is produced when phosphorus is heated in a stream of hydrogen. If the discharge was stopped, and the tube warmed, some of the gas absorbed was re-liberated. In this case, therefore, the absorption is due partly to occlusion, and partly to chemical action.

Sulphur absorbed well, providing the pressure of the gas was not below 0·5 mm. of mercury. Below this pressure the effect disappeared. When the contents of the tube were heated above the temperature of melting sulphur, and then allowed to cool, about one-sixth of the gas that had been absorbed was re-liberated. In this case, therefore, chemical action and occlusion account for the absorption.

The amounts of gas absorbed by iodine were similar to those found with phosphorus. There was no absorption when the pressure of the gas was below 1 mm. of mercury. About one-fifth of the gas which had been absorbed in an experiment was evolved when the contents of the tube were heated to 100°C.

5. Conclusions.

When considering the absorption of gases in the electric discharge tube, the part which the glass of the tube plays must be taken into account. Willows* has shown that in a Crooke's tube there is absorption of the gas, and that it is due probably to chemical action between the gases and the glass. He found that, with a volume of 108 c.c., and a pressure of 1 mm. of mercury, a current of $10{\cdot}8 \times 10^{-4}$ ampere passing for 30 minutes reduced the pressure nearly one-eleventh. This is a very small absorption compared with that found in the present work. The effect of absorption by the glass can thus be neglected in the present experiments. Willows and George† found that large quantities of hydrogen were absorbed by a quartz bulb, if the discharges were alternated with those in air. They supposed that chemical actions occur with the air, giving oxidation products; these are reduced by hydrogen. This would account for the large quantity of gas which can be absorbed. In the present work, as no oxygen was used, this effect is negligible. Thus practically all the absorption of gases shown in Tables I. and II. is attributed to the elements on the cathode.

The absorption of nitrogen is not an occlusion effect, but is due to chemical action. In several cases, the amount of gas absorbed is many times greater than the quantity of hydrogen liberated in the voltameter. This indicates that the phenomenon is not due to the positive ions within the tube, unless these ions are attached to neutral molecules of the gas. The nitrogen must assume an active modification when the discharge passes, and since the rate of absorption increases as the temperature falls, this modification must be intensified by cooling. Strutt‡ showed that pure nitrogen, subjected to the jar discharge, produces an active modification of nitrogen, and that it is not due to the ions in the discharge tube. He found that the modification was weakened by heating, and intensified by cooling. It acted on ordinary phosphorus and other elements.

Table I. shows that phosphorus, sulphur and iodine absorb nitrogen at a greater rate than other elements. In addition to the absorption by the cathode, there is chemical action occurring between the vapour of the elements and the gas.

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* "Phil. Mag.," 1 (1901).

† Proc. Phys. Soc., Vol, XXVIII. (1916).

‡ "Proc." Roy. Soc., Ser. A., 85 (1911).

This view is strengthened when we consider that the three substances have low boiling points. Some elements do not absorb nitrogen, and this is due, probably, to the absence of suitable conditions (as regards pressure and temperature) in the discharge tube.

Hydrogen is absorbed by sulphur, phosphorus and iodine. With this gas the effect appears to be due to two causes—occlusion of the gas within the substance, and chemical action. As the amount of gas occluded is very small, the more important factor in the absorption is chemical action. The results suggest that hydrogen assumes an active modification in the discharge tube, and that this modification is not due to gaseous ions. Recently, and while the present work was in progress, Wendt and Landauer* have shown that an active modification of hydrogen may be obtained by various methods, all dependent on gaseous ionisation; by α-rays from radium emanation, by electrical discharge under reduced pressure, and by the high-potential corona at atmospheric pressure. They state that this active hydrogen reduces sulphur, arsenic, phosphorus, mercury, nitrogen and both acid and neutral permanganates. It is unstable, and reverts to the ordinary form in about a minute. The activity is not due to gaseous ions, and they suppose that the stable hydrogen molecules undergo dissociation into free atoms, when an electron is removed during the process of ionisation; these atoms then attach themselves to neighbouring neutral molecules to form triatomic systems.

The disintegration of the cathode in a vacuum tube causes a film of metal to form on the adjacent glass. In this way a clean surface of metal is produced. When an electric discharge is passed, the conditions are favourable for the absorption of the gas by the metal. Considering the disappearance of the gas in a vacuum tube it is necessary to take into account the absorption of gas by the film of metal, and also the absorption by the glass walls of the tube. It is probable that the film of metal accounts for the greater part of the effect. Brodetsky and Hodgson showed that the disappearance of gases in vacuum tubes is a problem of some complexity, and that with metallic electrodes the effect is due principally to absorption of gas by disintegrated metal; it may also depend partly on the liberation of the alkali metals by electrolysis of the glass, and chemical combination of these metals with the gas.

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* "Am. Chem. Soc. Journ.," 42, May (1920).

DISCUSSION.

Dr. Goucher said that work bearing closely upon Mr. Newman's experiments on the disappearance of gas in the electric discharge had been carried on lately in the Research Laboratory of the General Electric Company. Some of it had already been published ("Phil. Mag.," XL., 585, Nov., 1920). It had led to conclusions precisely opposite to those of Mr. Newman. It did not appear to him that Mr. Newman had brought any direct evidence in favour of absorption being due to chemical action; the only direct evidence of that view would be the establishment of the "law of constant proportions," and of a constant ratio between the amount of gas disappearing and that of something else disappearing at the same time. The absorption of gas in the presence of phosphorus had been studied with especial care, and it had been found that there was no simple relation between the amount of gas disappearing and the amount of phosphorus used. Undoubtedly chemical action plays some part in the disappearance. For instance, the discharge through CO converts the gas into CO_2, and the relation between the current and the amount of gas transformed obeys accurately Faraday's law in suitable conditions. If, as in the discharge tubes which Mr. Newman used, ionisation is accompanied by recombination, it is not to be expected that Faraday's law should be obeyed. But even in such a case where a chemical action takes place, that action is not usually directly responsible for the disappearance of the gas. Chemical action will not cause complete absorption unless the products of the reactions are solid. Such compounds might be formed between nitrogen and the metal of the electrodes, but there is no evidence whatever that they can be formed between nitrogen and phosphorus or sulphur. The main cause of absorption of gas is, in the great majority of cases, adhesion to the walls of the vessel. This adhesion is greatly favoured by the deposition over the adhering gas of a solid layer of some other substance. The solid may be metal spluttered from the cathode or it may be, in the case of phosphorus, red phosphorus produced by the discharge from phosphorus vapour. All the work to which he referred indicated that actions of this kind were the main cause of the disappearance of gas in the electric discharge, except, possibly, when very active metals were used as cathodes.

Dr. H. Borns inquired whether formation of hydrides had been observed. If the alkali metals absorbed hydrogen also, as he understood the author to say, alkali hydrides might be formed, which were crystalline compounds like common salt, not in the least resembling metals. Quite recently Nernst and K. Moers ("Zeits. f. Elektrochem," Aug. 1, 1920) had electrolysed fused lithium hydride to see whether the hydrogen would really go to the anode in this case, as the metallic lithium would go to the cathode. Quantitatively the difficult experiments had not been conclusive, but there was evidence of hydrogen at the anode and hence experimental evidence confirming the possibility of the existence of both positive and negative hydrogen ions.

The Author, in reply to Dr. Goucher, said he did not think the results he mentioned were conclusive against chemical action. Lord Rayleigh had also attributed the effect to chemical action.

In reply to Dr. Borns, he could not say whether hydrides were formed or not.

IX. *Magnetic Separation of Neon Lines and Runge's Rule.*

By H. NAGAOKA, *Professor of Physics, Imperial University of Tokyo.*

RECEIVED DECEMBER 8, 1920.

ABSTRACT.

The Paper contains the results of an investigation of the Zeeman effect for Neon lines. The departures from Runge's Rule—that the magnetic separation of the lines are aliquot parts of the separation of the normal triplet—are discussed. It is concluded that such discrepancies are due to variations of the ratio e/m.

THE investigation of the Zeeman effect for neon lines by Lohmann* leads to the interesting result that the magnetic separations of most of the lines are aliquot parts of a normal triplet. This rule deduced by Runge† must, however, be looked upon only as an approximation, for most of the lines do not exactly conform to the rule.

Let e be the charge of an electron, m its mass at rest, H the magnetic field, and c the velocity of light, then the constant a given by

$$a=\frac{e}{m}\cdot\frac{1}{4\pi c}$$

should have a value $4{\cdot}698\times10^{-5}$ gauss^{-1} cm.$^{-1}$, corresponding to $\frac{e}{m}=1{\cdot}771\times10^{7}$ E.M.U. The last figure in a and $\frac{e}{m}$ may differ by small amounts according to different experiments. This is, however, not fulfilled in most of the neon lines; take, for instance, the lines 6717, 6266, 5852 and 5820, which are all assumed to be separated into triplets. The values of $10^5\times a$ for these lines as given by Lohmann are respectively 4·70, 4·40, 4·74 . . . ., while Takaminé and Yamada‡ give 4·82, 4·72, 4·84, 4·47 respectively for the above constant. As to the measurement of the field strength, the experiments of the last two mentioned authors were conducted with great care, the amount of separation of different lines being always compared with that of the helium line 6678. Thus the separation of the characteristic neon line 5852 is 1·5 per cent. greater, that of 5820 about 5 per cent. less than the normal value. As to the complex separations, if we express aliquot parts of a by p/q, the values of p and q are to some extent indefinite, as we can closely approximate to the rule by taking tolerably large numbers for p and q. The question is whether a is to be assumed as constant, or as variable within certain limits.

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* Dissertation, Halle (1907).

† "Phys. Zeit.," VIII., p. 232 (1907).

‡ "Proc." Tokyo Math. Phys. Soc., VII., p. 277 (1914).

Various reasons can be offered to explain the discrepancy. The theory of relativity would at once suggest that the discrepancy is a consequence of the high velocity of the radiating electrons, but to account for 5 per cent. difference, the velocity of electrons must be equivalent to a third part of the velocity of light, which is probably too great. In addition to this, relativity cannot account for the values of a smaller than the normal value. In a broad sense, we can attribute the difference of a from the normal value to the difference in the surrounding electromagnetic conditions, as the electrons in an atomic configuration are under mutual influence and also subject to attraction towards the nucleus. The inadequacy of the theories of the Zeeman effect may be the principal cause of these difficulties, but there are other doubts as to the experimental results from which these values are deduced.

In the first place, the amount of separation may not be proportional to the strength of the magnetic field, and hence may lead to discordant values. In the second place, the photographic exposure in former experiments was tolerably long, so that the uncertainty of the field strength by heating, the displacement of the echelon grating spectra even by small variations of temperature, may become sources of error in the photograms obtained by long exposure, sometimes extending to several hours.

In the present experiment, the photographic plate was sensitised by means of dicyanin, which made the plate more sensitive to red rays than panchromatic plates, so that the magnetically separated lines of neon in the yellow and red part of the spectrum can be photographed with comparatively short exposure, especially in strong fields. The constancy of magnetic field during this interval can be easily controlled, and by keeping the edge of the echelon grating horizontal, the red and yellow portion of the spectrum can be photographed on one plate, so that the separation of the lines can be compared generally for about 20 lines all in the same field. The line which is most approximate to the normal triplet and lies nearly in the middle of the said portion of the spectrum is 6266. The intensity of the line is neither too strong nor too weak; consequently the separation of the line can be accurately measured on all the plates, and was taken as the standard for gauging the separation of other lines. No exact measurement of the field was considered necessary, as the previous experiment of Takaminé and Yamada on 6266 gave the value $a=4{\cdot}72\times10^{-5}$ by comparison with the helium line 6678, the separation of which was accurately determined by Paschen and Back*. The line 5852 was not used as a standard as the deviation from the normal was greater than for 6266, and the intensity was too strong to admit of accurate determination of the mean position in plates for faint lines.

A neon tube having the gas at about 6 mm. pressure was placed between the poles of an electromagnet, having the pole pieces in the form of a double cone of semi-vertical angle of about 60 deg.; the gap between the pole pieces was about 7 mm., and the capillary portion of the tube was at the apex of the cones. As the tube was liable to crack at the electrodes and especially the capillary tube between the poles owing to local heating, the electrodes and the capillary tube were water-cooled. By this means, the tube in the mean field of 13 kilogauss was kept for more than 200 hours by passing discharge current from an induction coil with a Leyden jar of about 0·001 mfd. capacity in the circuit. The arrangement of water-jacketing the electrodes can be applied with advantage in other works with vacuum tubes, when it is required to run the tube for many hours with discharge current.

With the water-jacketing arrangement, the disintegration of the electrodes is greatly diminished. The capillary tube of the vacuum tube after long use turns brownish, and there is much absorption at the portion placed in strong field, that different parts of the capillary must be used to obtain good illumination. With one tube, the part long exposed to strong magnetic field was broken, and when examined by microscope was found to show a corroded appearance, ramified in several parts in the direction perpendicular to the field. As is evident from the intense light emitted from the portion so exposed, it appears that the ionised gas atoms move with great velocity and corrode the glass tube. A somewhat diffuse appearance of lines in strong fields is probably due to Doppler effect.

The light from the neon tube was focussed by a lens at the slit of the collimator of an echelon spectroscope with horizontal slit. After passing through the echelon grating of 35 plates, each of which was 0·935 cm. thick, the light was brought to a focus by an achromatic lens and the echelon spectrum was projected by a Rudolph's microplanar of 7·5 cm. focal length, on a collimator of a prism spectroscope, and then photographed with an objective of 10 cm. aperture and 75 cm. focal length. The plate was sensitised with dicyanin, which shortened the exposure more than any other sensitiser. At first the spectroscope was provided with a grating, but on account of the feebleness of light it was replaced by a 60 deg. prism of flint glass having faces of 5 cm. square. The spectrum generally extended from 6717 to 5852, and several exposures for the same field were taken. The magnetising current was kept constant by a regulating rheostat, and the pole pieces cooled by a steady stream of water circulating in the axis of the cones, which was perforated with the exception of the portion near the poles. Some of the spectrograms are shown in Plate I.

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* "Ann. d. Phys.," XXXIX., p. 897 (1912).

The spectrograms were measured by means of a micrometer screw specially polished and accurate to about 1μ, and provided with a microscope of low magnifying power. As the trace of the separated lines sometimes measured several tenths of a millimeter, it was unnecessary to push the measurements to the order of a micron. Referring the separation to that of 6266, the reduction can be easily effected and the strength of the field calculated from the known value of a for 6266.

The following table contains the value of $\frac{\delta\lambda}{\lambda^2 H}\cdot 10^{13}$, the types of separation by the author and by Takaminé and Yamada, the calculated value of $10^5 \times a$, and the value of $\frac{e}{m}10^{-7}$ calculated from a. The bracketed values of px and of $\frac{e}{m}10^{-7}$ refer to results obtained from the normal components only.

The comparison of the types of separation will show the indefiniteness of Runge's rule for complex separation. The deviations from the rule are usually large for parallel components, for which the displacement is tolerably small. They may be due either to the smallness of the displacements or to the peculiarity of the parallel components. The normal components are more regular, and most of the deviations may be ascribed to errors of measurements. There is some doubt, if it is not proper in some cases to use the different values of q in the calculation of $10^5 \times a$ for the normal and the parallel components.

In the table, the values of px are more approximate to the observed values for the normal than for the parallel components. The values found from the normal components only for complex separation are given in brackets for the sake of comparison, as likewise the values of $\frac{e}{m}$ derived from the same numbers, while those obtained from Takaminé

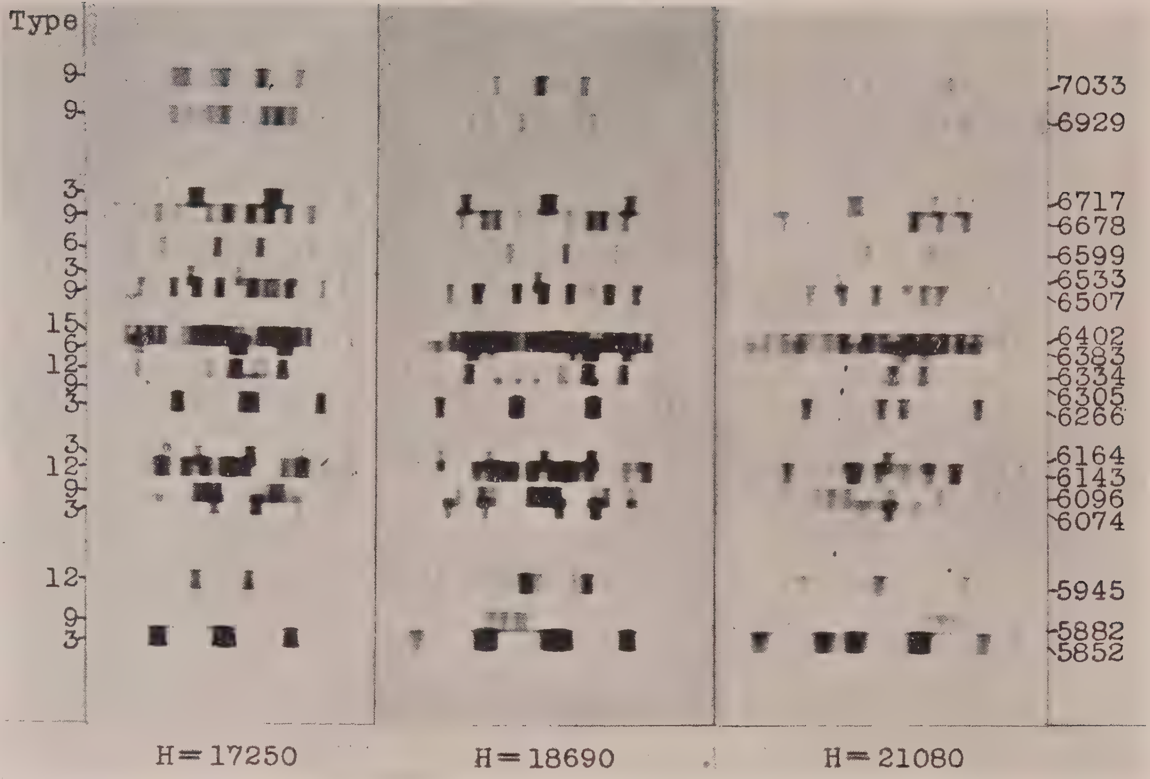

Plate I.

[*To face* p. 86.

λ in Å.U.		$\frac{\delta\lambda}{\lambda^2 H}10^{13}$	Type (T.Y.)	Multiples of x.	$e/m.10^{-7}$	(T.Y.)	(L.)
6717......	p	0	0	......	1·821	(1·816)	(1·772)
	n	4·83	a				
6678......	p	0	0	......	1·774	(1·857)	(1·756)
	p	1·23	$\frac{3}{12}a\left[\frac{1}{4}a\right]$	$3x=1\cdot172$			
	n	5·09	$\frac{13}{12}a\left[\frac{4}{4}a\right]$	$13x=5\cdot079$ (5·079)	(1·768)		
	n	6·27	$\frac{16}{12}a\left[\frac{5}{4}a\right]$	$16x=6\cdot251$ (6·251)			
	n	7·40	$\frac{19}{12}a\left[\frac{6}{4}a\right]$	$19x=7\cdot423$ (7·423)			
6599......	p	1·47	$\frac{4}{13}a$	$4x=1\cdot432$	1·760	(1·755)	(1·785)
	n	5·02	$\frac{14}{13}a$	$14x=5\cdot012$ (5·005)	(1·754)		
	n	6·43	$\frac{18}{13}a$	$18x=6\cdot444$ (6·435)			
6533......	p	0	0	......	1·831	(1·802)	(1·769)
	n	3·24	$\frac{2}{3}a$				
6507......	p	0	0	......	1·798	(1·769)	(1·769)
	p	1·57	$\frac{5}{15}a$	$5x=1\cdot590$			
	n	3·84	$\frac{12}{15}a$	$12x=3\cdot816$ (3·816)	(1·800)		
	n	5·42	$\frac{17}{15}a$	$17x=5\cdot406$ (5·406)			
	n	6·98	$\frac{22}{15}a$	$22x=6\cdot996$ (6·996)			
6402......	p	0	0		1·766	(1·774)	(1·727)
	p	0·80	$\frac{1}{6}a$	$x=0\cdot780$			
	p	1·59	$\frac{2}{6}a$	$2x=1\cdot560$			
	n	4·69	$\frac{6}{6}a$	$6x=4\cdot680$ (4·680)	(1·763)		
	n	5·45	$\frac{7}{6}a$	$7x=5\cdot460$ (5·460)			
	n	6·25	$\frac{8}{6}a$	$8x=6\cdot240$ (6·240)			
	n	7·02	$\frac{9}{6}a$	$9x=7\cdot020$ (7·020)			
	n	7·78	$\frac{10}{6}a$	$10x=7\cdot800$ (7·800)			
6383......	p	3·82	$\frac{9}{11}a\left[\frac{10}{13}a\right]$	$9x=3\cdot840$	1·765	(1·782)	(1·724)
	n	2·94	$\frac{7}{11}a\left[\frac{9}{13}a\right]$	$7x=2\cdot986$ (2·990)	(1·767)		
	n	6·86	$\frac{16}{11}a\left[\frac{19}{13}a\right]$	$10x=6\cdot826$ (6·834)			

λ in Å.U.		$\frac{\delta\lambda}{\lambda^2 H}10^{13}$	Type (T.Y.)	Multiples of x.	$e/m.10^{-7}$	(T.Y.)	(L.)
6334......	p	1·76	$\frac{3}{8}a\left[\frac{4}{11}a\right]$	$3x=$ 1·777	1·785	(1·814)	(1·752)
	p	3·50	$\frac{6}{8}a\left[\frac{8}{11}a\right]$	$6x=$ 3·554			
	n	3·50	$\frac{6}{8}a\left[\frac{8}{11}a\right]$	= 3·554 (3·560)	(1·789)		
	n	5·41	$\frac{9}{8}a\left[\frac{12}{11}a\right]$	$9x=$ 5·331 (5·340)			
	n	7·15	$\frac{12}{8}a\left[\frac{16}{11}a\right]$	$12x=$ 7·108 (7·120)			
	n	8·86	$\frac{15}{8}a\left[\frac{20}{11}a\right]$	$15x=$ 8·885 (8·900)			
6305......	p	0	0		1·794	(1·742)	(1·637)
	p	1·08	$\frac{2}{9}a\left[\frac{1}{4}a\right]$	$2x=$ 1·056			
	n	4·75	$\frac{9}{9}a\left[\frac{4}{4}a\right]$	$9x=$ 4·752 (4·752)	(1·792)		
	n	5·81	$\frac{11}{9}a\left[\frac{5}{4}a\right]$	$11x=$ 5·808 (5·808)			
	n	6·87	$\frac{13}{9}a\left[\frac{6}{4}a\right]$	$13x=$ 6·864 (6·864)			
6266......	p n	0 4·720	0 a		1·779	(1·779)	(1·657)
6217......	p	0	0	......	1·772	(1·780)	(1·767)
	p	3·86	$\frac{9}{11}a\left[\frac{5}{6}a\right]$	$9x=$ 3·840			
	n	......	... $\left[\frac{4}{6}a\right]$	......			
	n	7·22	$\frac{17}{11}a\left[\frac{9}{6}a\right]$	$17x=$ 7·253 (7·253)	(1·769)		
	n	11·12	$\frac{26}{11}a\left[\frac{14}{6}a\right]$	$26x=$11·093 (11·093)			
6164......	p n	0 6·35	0 $\frac{4}{3}a$	......	1·796	(1·823)	(1·714)
6143......	p	1·32	$\frac{3}{11}a\left[\frac{3}{12}a\right]$	$3x=$ 1·253	1.738	(1·849)	(1·835)
	p	2·57	$\frac{6}{11}a\left[\frac{6}{12}a\right]$	$6x=$ 2·506			
	n	4·60	$\frac{11}{11}a\left[\frac{11}{12}a\right]$	$11x=$ 4·595 (4·591)	(1·731)		
	n	5·82	$\frac{14}{11}a\left[\frac{14}{12}a\right]$	$14x=$ 5·848 (5·843)			
	n	7·10	$\frac{17}{11}a\left[\frac{17}{12}a\right]$	$17x=$ 7·101 (7·095)			
	n	8·34	$\frac{20}{11}a\left[\frac{20}{12}a\right]$	$20x=$ 8·357 (8·347)			

λ in Å.U.		$\frac{\delta\lambda}{\lambda^2 H}10^{13}$.	Type (T.Y.)	Multiples of x.	$e/m.10^{-7}$	(T.Y.)	(L.)
6096......	p	0	0		1·755	(1·771)	(1·769)
	p	0·84	$\frac{3}{16}a\left[\frac{1}{7}a\right]$	$3x=$ 0·875			
	n	5·28	$\frac{18}{16}a\left[\frac{8}{7}a\right]$	$18x=$ 5·250 (5·250)	(1·759)		
	n	6·13	$\frac{21}{16}a\left[\frac{9}{7}a\right]$	$21x=$ 6·125 (6·125)			
	n	6·95	$\frac{24}{16}a\left[\frac{10}{7}a\right]$	$24x=$ 7·000 (7·000)			
6074......	p	0	0		1·742	(1·734)	(1·709)
	n	6·93	$\frac{3}{2}a$				
5976......	p	0	0		1·794	(1·771)	(1·759)
	p	2·47	$\frac{1}{2}a$	$x=$ 2·370			
	n	—	— [a]	———			
	n	7·07	$\frac{3}{2}a$	$3x=$ 7·110 (7·095)	(1·785)		
	n	9·49	$\frac{4}{2}a$	$4x=$ 9·480 (9·460)			
5945......	p	1·08	$\frac{3}{15}a\left[\frac{2}{10}a\right]$	$3x=$ 0·953	1·804	(1·782)	(1·763)
	p	1·92	$\frac{6}{15}a\left[\frac{4}{10}a\right]$	$6x=$ 1·906			
	n	5·12	$\frac{16}{15}a\left[\frac{11}{10}a\right]$	$16x=$ 5·082 (5·077)	(1·796)		
	n	6·03	$\frac{19}{15}a\left[\frac{13}{10}a\right]$	$19x=$ 6·035 (6·029)			
	n	7·00	$\frac{22}{15}a\left[\frac{15}{10}a\right]$	$22\ x=$ 6·988 (6·981)			
	n	7·89	$\frac{25}{15}a\left[\frac{18}{10}a\right]$	$25x=$ 7·941 (7·933)			
5882......	p	0	0		1·781	(1·737)	(1·700)
	p	0·77	$\frac{1}{6}a\left[\frac{2}{12}a\right]$	$x=$ 0·788			
	n	6·32	$\frac{8}{6}a\left[\frac{17}{12}a\right]$	$8x=$ 6·304 (6·304)	(1·782)		
	n	7·08	$\frac{9}{6}a\left[\frac{19}{12}a\right]$	$9x=$ 7·092 (7·092)			
	n	7·88	$\frac{10}{6}a\left[\frac{21}{12}a\right]$	$10x=$ 7·880 (7·880)			
5852......	p	0	0		1·843	(1·824)	(1·786)
	n	4·89					

and Yamada's and Lohmann's measurements are also given side by side. From these results, we find that $\frac{e}{m}$ obtained from n-components are somewhat nearer to the normal value. Some of the values of p and q are artificial, especially when they are large. Thus different values of p and q were given by the experimenters. In addition to this, we sometimes meet with doubtful cases, such, for example, as the separation of 5852; it is here simply taken equal to a, but if large values of p and q are admissible, they may be 23 and 22 respectively, which exactly corresponds to the normal value. This is probably inadmissible, and many cases of complex separation seem likely to have been made to conform to Runge's rule, by assuming large values of p and q. The physical reasons for taking these values are not yet clear, and must be considered only as empirical, so that much weight cannot be given to these numbers. The good coincidence of some of the components is indeed wonderful, and the theory of the Zeeman effect will in the future be able to elucidate the present problem, which must have an intimate connection with the configuration of electrons in the atomic structure.

The question, whether the separation is proportional to the field or not, was examined on about 70 photograms, with the result that between the fields of 8500 to 21500 gauss the linear relation holds true.

The mean value of $10^{-7}\,\frac{e}{m}$ found for the same range of fields from p and n components of 19 different lines is 1·784, and from n-components only 1·782. As the value of the standard line is still subject to some experimental error, I cannot place much weight on the absolute value, but it is quite clear that the mean value nearly corresponds to that of the normal triplet.

The inspection of the table will at once show that the value of $\frac{e}{m}$ is not a constant, which at first sight seems a little paradoxical. From the point of view of the electron theory, the discrepancies in the value of $\frac{e}{m}$ may arise from various causes, which affect the electron configuration in the atom. The value of $\frac{e}{m}$ usually given in the electron theory is deduced on the assumption that an electron is at rest or moves slowly in an infinite space without any external electromagnetic disturbance. This condition is never satisfied in the atomic configuration. If the speed of the radiating electrons is comparable with that of light, the correction due to relativity will evidently enter into the result, even if the electron be moving singly with uniform motion. In the case of neon, it may be argued that the radiating electron cannot satisfy the said condition, so that the mass of the electron would be different from that of the electromagnetic mass usually treated in the electron theory. The case of electrons moving at a fixed distance from each other was treated by Nicholson,* from which it is evident that the mass of two or more electrons moving together is greater than that of electrons taken separately; the consequence is that the value of $\frac{e}{m}$ should be generally smaller than the normal value, which is approximately $1 \cdot 77 \times 10^7$ E.M.U. This is fulfilled in many of the lines above cited, and may either be ascribed to the relativity correction or to the motion of the electron groups. Perhaps the latter explanation is the more probable. In some cases the value of $\frac{e}{m}$ is greater than the normal value, and some explanation must be given.

The effective magnetic field H is assumed to be exactly the same as that given by the electromagnet. If we admit the existence of rings of electrons rotating about the nucleus, the above assumption cannot be justified. The action of the external magnetic field will be superposed on the atomic field, and what is felt in the Zeeman effect will be the resultant of these fields. That the value of $\frac{e}{m}$ is greater or less than

the discrepancy in $\frac{e}{m}$ must be traced to other causes.

the normal value may imply the fact that the effective field is smaller or greater than the external. It is difficult to estimate the magnitude of the atomic field, so that the value of H in deducing Runge's formula is different from that of the electromagnet. If this is really the case, the linear relation of the amount of separation with the strength of the field will be but approximately fulfilled, so that the curve representing the above is no longer straight. As above stated, the deviation from linearity was within experimental error. Whether the field strength is not great enough to bring out the effect, or the atomic field is negligibly small compared with the external field, is rather difficult to answer. In the latter case,

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* Proc. Phys. Soc., Vol. XXVII., p. 217 (1915).

If we consider the effect of the magnetic field on the radiation as due to the minute change of configuration in the electronic configuration within the atom, the quasi-elastic force acting on the radiating electron must be subject to a change, which may not be too small to detect by means of the delicate spectral test. The effect was examined for triplets and nonets, but with no positive result. This important problem remains still unsolved for the most of lines, but if the change is appreciable, it will lead to important consequences as regards electronic configuration within the atom. The method similar to the examination of the so-called pole effect will lead to the final solution of the problem.

The electronic configuration will be slightly altered by the external magnetic force, and the action of the electric field on the electrons will be correspondingly affected. This will account for the alteration of quasi-elastic force acting on the vibrating electrons, but the atomic electric field may act somewhat like the Stark effect, and cause the displacement of the lines. Probably the change in the configuration will not cause disturbances in the electric field as to produce appreciable displacement, except for anomalous magnetic separations as observed in the mercury line 5790, the satellites of the helium lines 5876 and 4713, and the oxygen triplets in the ultra-violet.

As to the complex separations of the neon lines, the theory of coupling by Voigt can explain nearly all the types here given. The sextets can be ascribed to two coupled electrons, the nonets to three, and so on; and if we consider the radiating electrons as moving together, the value of $\frac{e}{m}$ will evidently be open to question, as in the problem proposed by Nicholson.

The formula for the mass of coupled electrons shows an increase, so that $\frac{e}{m}$ must decrease. For the line 6402, which is separated into 15 components, $\frac{e}{m}$ is very near to the normal value, and shows no decrease in spite of the number of coupled electrons. This seems at first sight contradictory, but various causes may be acting simultaneously, so that in the above example m may increase, but the effective field may decrease, so that different effects compensate each other and give the normal value to $\frac{e}{m}$.

From the mechanical analogy of the Zeeman effect, it seems quite likely thatt he sextet corresponds to the case of two equal roots of the equation giving the natural frequency of the vibrating electron, and nonets to three equal roots, and similarly for other complex separations. Tentative experiments were tried to see if the lines giving rise to sextets, nonets, &c., are not multiple lines having minute differences among themselves. This was tested by bringing the high resolving power of the Fabry-Perot interferometer having thick air plate to bear on the neon lines, but all the lines were apparently simple and showed no signs of multiplicity.

According to Aston,* neon consists mostly of isotopes of atomic weight 20 and 22, while an isotope intermediate between the two may exist in small quantity. Each of the isotopes will give rise to spectrum lines which are nearly coincident. The constant difference in the frequency of lines, of which there are two very definite values, according to Burns, Meggers and Merrill,† seems too large to account for the isotopes, but the complexity of the Zeeman effect in neon may have some connection with the existence of isotopes, and especially the observed fact that the types of separation are all multiples of three.

The various causes above cited will, to some extent, affect the displacement of the lines due to magnetic separation, but we have at present no means of discriminating and assigning the discrepancies to these causes.

A more thorough study of the atomic structure will be necessary to give weight to these different causes, but there may be other causes not yet mentioned, in which the shape of electrons forms a principal feature. The supporters of ring-shaped electrons may find some clue to the elucidation of Runge's rule, and the discrepancies to which it is subject. One of the departures in this direction is the indeterminateness of the effective magnetic field, for the ring electron has its own field, and by making gyrostatic motion about its own axis, the field may be increased or decreased.

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* "Phil. Mag.," Vol. XXXIX., p. 449, 611 (1920).

† "Bull." Bureau Stand., Vol. XIV., p. 765 (1919).

Another suggestion is that, if the ring be contractile, the electro-magnetic mass may change, consequently different values of $\frac{e}{m}$ are to be expected, as already indicated by the experiment. Notwithstanding the various aspects under which the assumption of a ring electron is capable of solving the present problem, we have as yet no definite idea to account for the various types of separation, which is so characteristic of the neon spectrum.

Borrowing a mechanical analogy, we can consider a ring electron as a vortex ring in an incompressible liquid. The circulation about the ring corresponds to magnetic flux, and the vortex to the rapidly moving charge. In this aspect the ring electron is no more than the vortex atom electronised. Such vortex rings are susceptible of radial vibrations and subject to flutings. The Zeeman effect is evidently analogous to flutings caused by the change of circulation about the ring, and is due to small changes of frequency by the magnetic flux in the circuit of the ring electron. We must, however, suppose that such flutings are controlled by the central nucleus, and the types of magnetic separation will be determined by the atomic configuration of the electrons.

Other singular properties of neon may have some connection with the present investigation, but as no thorough study of the separation of lines in the more refrangible part of the spectrum has as yet been made, any hasty conclusion from the investigation of the red and yellow lines may lead to erroneous conclusions.

It will not be out of place to give a mechanical analogy of small oscillations for illustrating the Zeeman effect and Runge's rule. Following Lorentz,* suppose the displacements of the electrons in a radiating atom to be determined by the normal co-ordinates, p_h, so that the kinetic energy, T, and the potential energy, V, are given by

$$T=\tfrac{1}{2}\sum_{1}^{n} A_h \dot{p}_h^2$$

$$V=\tfrac{1}{2}\sum_{1}^{n} B_h p_h^2.$$

By the action of the magnetic field, the system will be slightly disturbed, and the normal co-ordinates cease to be exactly normal; consequently the energies in the disturbed system will be given by

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* "Ann. d. Phys. u. Chem.," LXIII., p. 278 (1897); Theory of Electrons (1909).

$$T=\tfrac{1}{2}\,\Sigma(A_h+a_h)\dot{p}_h{}^2+\Sigma a_{hk}\dot{p}_h\dot{p}_k,$$
$$V=\tfrac{1}{2}\,\Sigma(B_h+b_h)p_h{}^2+\Sigma b_{hk}p_hp_k,$$

where h is different from k, a's and b's are small, compared with A's and B's. Further we suppose that a's and b's are proportional to the magnetic field. The new force brought into play by the action of the magnetic field is given by the vector product of the velocity of the electron into the magnetic force. The force acting on the electrons by applying the magnetic field is given by

$$P_h=\Sigma c_{hk}\dot{p}_k.$$

By the condition that P_h is perpendicular to $\dot{p}_h$,

$$c_{hk}=-c_{kh}.$$

Thus the equations of motion, of which there are n, are of the general form

$$(A_h+a_h)\ddot{p}_h+\Sigma a_{hk}\ddot{p}_k+(B_h+b_h)p_h+\Sigma b_{hk}p_k-\Sigma c_{hk}\dot{p}_k=0.$$

Put $p_h=u_he^{i\nu t}$, and find the equation for determining the frequency ν. By substituting in the equation of motion, we get n equations of the form

$$\{(B_h+b_h)-\nu^2(A_h+a_h)\}u_h-i\nu\,\Sigma c_{hk}u_k+\Sigma(b_{hk}-\nu^2a_{hk})u_k=0.$$

Eliminating $u_1, u_2, \ldots, u_n$, we obtain the determinantal equation for finding the frequencies

$$\begin{vmatrix} (B_1-\nu^2A_1)+(b_1-\nu^2a_1), & -i\nu c_{12}+b_{12}-\nu^2a_{hk}, & \ldots \\ -i\nu c_{21}+b_{21}-\nu^2a_{21}, & (B_2-\nu^2A_2)+(b_2-\nu^2a_2), & \ldots \\ \ldots\ldots, & \ldots\ldots & \end{vmatrix}=0$$

It is to be remarked that terms in small letters are very small compared with those given by the capital letters.

In the undisturbed system, let us suppose that a certain number, say the first k natural frequencies $\sqrt{\frac{B_h}{A_h}}$ $(h=1, 2, 3 \ldots k)$ have a common value N, and let us seek values of ν satisfying the above condition and nearly equal to N. In that case, we can suppose $N^2-\nu^2$ to be a small quantity nearly equal to $2N\delta N$, where δN is the difference of N from ν, so that the additional small quantities in the determinant cannot be generally neglected, but owing to the presence of the factor, N, $2N\delta N$ is large compared with δN, which is very small.

Dividing the determinant by A_1, A_2, . . . , and putting $\frac{a}{A}=\alpha$, $\frac{b}{A}=\beta$, we find that the result is approximately given by

$$(N^2_{k+1}-\nu^2)(N_{k+2}-\nu^2)\ \ldots\ (N^2_n-\nu^2)\ .\ \Delta=0,$$

where

$$\Delta=\left|\begin{array}{lll} N^2_1-\nu^2+\beta_1-\nu^2\alpha_1, & -i\nu\gamma_{12}+\beta_{12}-\nu^2\alpha_{12}, & \ldots \\ -i\nu\gamma_{21}+\beta_{21}-\nu^2\alpha_{21}, & N^2_2-\nu^2+\beta_2-\nu^2\alpha_2, & \ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \end{array}\right|$$

For finding the frequencies little different from the common value

$$N_1=N_2=\ldots\ldots=N_k=N\ ;$$

the determinantal equation changes to

$$\Delta=\left|\begin{array}{llll} N^2-\nu^2+\lambda_1, & \mu_{12}, & \mu_{13}, & \ldots \\ \mu_{21}, & N^2-\nu^2+\lambda_2, & \mu_{23}, & \ldots \\ \mu_{31}, & \mu_{32}, & N^2-\nu^2+\lambda_3, & \ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \end{array}\right|=0,$$

where for simplicity

$$\lambda_1=\beta_1-\nu^2\alpha_1,\ \ldots$$
$$\mu_{12}=-i\nu\gamma_{12}+\beta_{12}-\nu^2\alpha_{12},\ \ldots$$

Evidently α_{hk} and β_{hk} give rise to the disturbances in the normal co-ordinates of the initial system. We suppose the change due to the magnetic field to be very small; then α_{hk} and β_{hk} will be generally small compared with $N^2-\nu^2$, so that $\mu_{hk}=-i\nu\gamma_{hk}$ nearly, and is proportional to the magnetic field.

In $\lambda_h=\beta_h-\nu^2\alpha_h$, we remark that β_h is of the same order of small quantities as δN, and most probably $\nu^2\alpha_h$ would be of similar order. This point is still questionable, as ν^2 is a large quantity. Under the above supposition the equation turns out to be

$$\left|\begin{array}{llll} N^2-\nu^2 & \mu_{12} & \mu_{13} & \ldots \\ \mu_{21} & N^2-\nu^2 & \mu_{23} & \ldots \\ \mu_{31} & \mu_{32} & N^2-\nu^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{array}\right|=0,$$

which is a skew determinant, since

$$\mu_{hk}=-\mu_{kh}.$$

The determinant gives rise to an equation of the form

$$(N^2-\nu^2)^k+C_2(N^2-\nu^2)^{k-2}+C_4(N^2-\nu^2)^{k-4}+\ .\ .\ .=0$$

If $\nu^2\alpha_h$ is not negligibly small, the right-hand side is equal to a small quantity, which is nearly constant. When the position of the lines remains unaltered, $\nu^2\alpha_h$ must be very small, and the change in kinetic energy due to magnetic force is negligible. This condition is nearly satisfied in the neon spectrum. The above consideration therefore shows that the change in kinetic and potential energies by the action of the external magnetic force of the electromagnet cannot be detected by the experiments.

When k is odd, the last term contains $N^2-\nu^2$, giving the root $N=\nu$, which shows that the initial position of the line remains unaltered. By cancelling the common factor $N^2-\nu^2$, we obtain an equation which holds for the case of an even number of equal roots. If the roots for natural frequencies in the disturbed system are of the form

$$N^2-\nu^2=\pm R_1,\ \pm R_2,\ \pm R_3,\ .\ .\ .\ ,$$

we get by expansion

$$\nu_h=N\pm\frac{R_h}{2N},$$

where R_h is directly proportional to the magnetic field to a first approximation. Thus the mechanical analogy of the magnetic separation of the neon lines is demonstrated. I remark that the separation is symmetrical about the initial position, as has already been observed, indicating the evanescence of the influence of the magnetic field, since the effect of a_h and b_h affected with H is immeasurably small.

Whether the external magnetic force is not strong enough to bring out the expected result, or the resolving power of the interferometers is not high enough to separate the closely packed lines into the components, awaits further experiments.

From these considerations, we find that the change in frequency depends principally on c_{hk} and hence on γ_{hk}. Since this quantity arises from the force $\frac{e}{c}$[velocity $\times H$], the bracket signifying that it is a vector product, it is evident that γ_{hk} is proportional to $\frac{eH}{4\pi mc}$, which may be written aH, following the notation at the outset. Experiment further shows that the roots R_1, R_2, . . . are proportional to the field. In the case of complex separation with the electric force either parallel or normal to the magnetic field, we find the relation that the differences between the roots of the determinantal equation are equal, or $R_2 - R_1 = R_3 - R_2 = \ldots$, which is again an aliquot part of the separation of a normal triplet. It can only be conjectured that the rule is a consequence of some quantum relation.

How far Runge's rule holds true is still a problem, but the present experiments and mechanical analogies show that small terms of correction are generally negligible except γ_{kh}, depending on the field and the ratio $\frac{e}{m}$. Thus the result is but approximate, and probably no exact coincidence in the values of $\frac{e}{m}$ can be expected, though further accuracy in the measurements can be attained. From the preceding discussion, it is also evident that the change in atomic configuration by an external magnetic force of 20 kilogauss is not sufficient to affect appreciably the kinetic and potential energies of the system.

The present investigation was made at the Institute for Physical and Chemical Research with the view of utilising the magnetic separation of neon lines for surveying heterogeneous magnetic fields. The photograms were mostly taken by Mr. T. Mishima and Mr. T. Okazawa, and the micrometric measurements and reductions were made by Mr. S. Sakurai, all assistants in the Institute, to whom my best thanks are due.

DISCUSSION.

Prof. J. W. NICHOLSON said the Paper was of the very greatest value. The experimental results were of high importance. He did not, however, think that the explanation of the discrepancies advanced by the author was adequate. Personally, he did not believe in the so-called series electron, *i.e.*, an electron very much more remote from the nucleus than any others in the atom. The modern conception of the normal helium atom assumed that one electron pursued a nearly circular elliptic orbit at one focus of which the nucleus $2e$ was situated. The orbit of the other electron was a smaller and more elongated ellipse intersecting the first. The arrangement was such that although the second electron would sometimes be quite near the nucleus, at other times it could be as far away as the first, and the simple theory of the series electron could not be expected to hold. The effect of a magnetic field was to change the character of the orbits. There were various possibilities. He had worked out a possible case for helium some years ago. It consisted of two concentric circular orbits, the electrons moving in such a way as to keep as far apart as possible. The ratio of the radii of these orbits was about 9·8 : 1. He felt sure that the magnetic separation did not depend only on e/m, but was also a function of this ratio of the orbits, and that values such as he had indicated would probably put right the discrepancies referred to in the Paper.

Mr. M. N. Saha (of Calcutta University) said, in regard to Prof. Nicholson's remarks, that the difficulty in extending Bohr's theory to the problem of helium radiation was that a complete solution had not yet been obtained of the problem of three bodies. Poincaré had succeeded in giving a solution assuming the masses of two of them to be infinitesimally small compared with that of the central body. He had tried to apply Poincaré's method to the helium atom, but so far without success. With regard to the size of the orbits, he referred to a Paper by Mr. S. N. Basu in the "Phil. Mag.," November 1920. He deduced Rydberg's Law from Bohr's theory by assuming the central attracting system to be composed of a net unit positive charge plus an electric doublet of definite moment. The fact that the ordinary lines of an element could be grouped into series with the Rydberg constant N showed clearly that the lines are produced by the quantum changes of orbit of the series electron, the nuclear charge and the remaining electrons merely determining the central attracting system. The size of the natural orbits of the electrons could be deduced from a corollary of Basu's theory, viz., that the radius is inversely proportional to the ionisation potential. For helium this is 25·4 volts, and, taking the natural radius of the electron in the H-atom as $0{\cdot}532 \times 10^{-8}$ cm., the radius of the outer electron of the He-atom is $0{\cdot}28 \times 10^{-8}$ cm. When this electron is lost the next one describes a circular orbit of $0{\cdot}134 \times 10^{-8}$ cm. (taking the second step ionisation potential as 54·6 volts), the ratio being about 2 : 1. In the neutral atom the orbit of the inner electron will be pushed inwards, but he could not see how the ratio could ever be so high as 9 to 1, as suggested by Prof. Nicholson.

Prof. Rankine asked Prof. Nicholson how much the ratio of the orbits differed before and after radiation took place.

Prof. Nicholson replied at length to the various questions raised. In the type of orbit he had mentioned the ratio was never less than about 7. As regards the Rydberg constant necessitating the series electron being far away he pointed out that when this was so they obtained the Balmer formula involving N/m^2. If it was less far away this was replaced by $N/(m+\mu)^2$, and if still less by $N/(m+\mu+\frac{\alpha}{m^2})^2$. All these corrections to the denominator were simply in order to preserve the constant N in the numerator. They could get equally satisfactory series formulæ by introducing the corrections in the numerator instead.

X. *A Method of Demonstrating the Retroactive Property of a Triode Oscillator. By* E. V. Appleton, *M.A., M.Sc.*

Received September 24, 1920.

ABSTRACT.

The author, following Vallauri, gives an approximate treatment of the conditions which gives rise to retroaction between the grid and anode circuits of a triode valve, and describes an arrangement of circuits whereby the property can easily be demonstrated to a large audience.

The method of using triode retroaction for the amplification of wireless signals is now well known. Of the various examples of the use of this method the arrangement described by Armstrong[*] may be regarded as typical. Here the usual oscillatory circuit L_1C (*see* Fig. 1) is connected to the grid and filament of a triode, retroaction being obtained by means of the coil L_2 in the anode circuit.

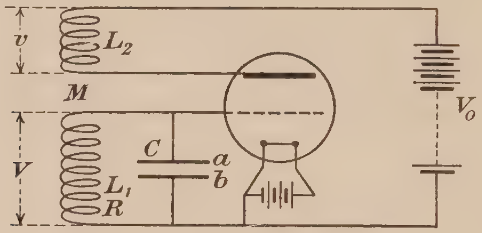

Fig. 1.

If the mutual inductance between L_1 and L_2 is of the correct sense and magnitude it is possible to cause the variations of the anode current, produced by a free or forced oscillation in L_1C, to induce favourably timed electromotive forces in the oscillatory circuit and so partly or wholly compensate for Joulean and other losses. In the case of total compensation, of course, the oscillation is undamped.

The following approximate quantitative treatment, suggested by the work of Vallauri,[†] seems to make the matter clearer. Consider a free oscillation in the circuit L_1C.. Ignoring for the moment the effect of L_2 we have for the ordinary expression of the electromotive forces in the circuit

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[*] E. H. Armstrong, "Proc." Inst. Rad. Eng., 3, p. 215, 1915.

[†] G. Vallauri, "L'Elettrotecnica," 3, p. 43, 1917.

$$L_1\ddot{Q}+R\dot{Q}+\frac{Q}{C}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where Q is the charge on the plate (a) of the condenser at any instant.

If we now take into account the mutual inductance M between L_1 and L_2, and also assume that the resistances of both coils are small compared with their reactances for the particular frequency considered, we have approximately

$$\frac{V}{v}=\frac{M}{L_1},$$

where V and v are the induced anode and grid voltages respectively. Also the value of v at any instant is $\frac{Q}{C}$.

If we now assume that the anode current changes in the tube are proportional to the anode and grid voltage changes involved, we have, if I is the value of the anode current,

$$I=av+bV+c,$$

where a, b and c are constants.

On substituting for V and v this becomes

$$I=a\frac{Q}{C}+\frac{bMQ}{L_1C}+c,$$

that is

$$M\dot{I}=\frac{aM\dot{Q}}{C}+\frac{bM^2\dot{Q}}{L_1C}.$$

Now $-MI$ is the value of the E.M.F. induced in the oscillation circuit by the anode current variations. In the retroactive case we must, therefore, rewrite (1) as follows :

$$L_1\ddot{Q}+\left(R+\frac{Ma}{C}+\frac{bM^2}{L_1C}\right)\dot{Q}+\frac{Q}{C}=0 \quad . \quad . \quad . \quad . \qquad (2)$$

For retroaction in the circuit we are considering it is well known that M must be negative and thus the effective resistance of the circuit may be written as

$$R-\frac{Ma}{C}+\frac{M^2b}{L_1C},$$

where M represents the numerical value of the mutual inductance. We may make this effective resistance as small as we please by increasing the numerical value of M, as in a practical case $\frac{bM}{L_1}$ is small compared with a. This reduction in the effective resistance of the oscillatory circuit makes, of course, a corresponding reduction in the damping. Now the time required for a free oscillation to decay to one-hundredth of its value in a non-retroactive circuit is

$$\frac{9\cdot212L_1}{R}\text{ seconds}$$

(L and R being in a consistent set of units). In a retroactive circuit we must write this

$$\frac{9\cdot212L_1}{R-\frac{Ma}{C}+\frac{bM^2}{L_1C}}\text{ seconds.}$$

It is possible to demonstrate the difference between these two times in circuits where L_1 is very large and the signals are of acoustic frequency. A simple circuit for this is shown in Fig. 2. The condenser C is charged by putting the change-over switch in the (A) position and then discharged through the inductance L_1 by changing to the (B) position. The oscillation in the L_1C circuit is heard in the telephone T. By performing this experiment first with the coils L_1 and L_2 very loosely coupled, and afterwards with the value of M adjusted so that the effective resistance is appreciably diminished, it is possible to show to a large audience that a free oscillation, which, without retroaction, sinks to inaudibility in a fraction of a second, can, by means of retroaction, be prolonged for 4 or 5 seconds.

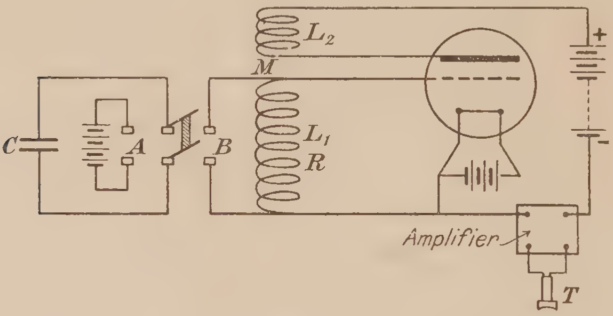

Fig. 2.

If a vibration galvanometer is used instead of a telephone a quantitative determination of the effective resistance in any particular case is possible.

[*Note added Jan. 26th.*—A slightly more satisfactory arrangement than that shown in Fig. 2 is obtained when the oscillatory circuit is impulsed electrically by means of a neighbouring aperiodic circuit.]

DISCUSSION.

Prof. W. Eccles said the method would be very useful as a lecture experiment. Those who were accustomed to high-frequency work were familiar with these phenomena, but students were often mystified by the perfect silence of high-frequency circuits. The author's arrangement for demonstrating the effect at audio frequencies was therefore valuable. It reminded him of one apparatus, which Mr. Jordan and himself had shown, in which a tuning fork took the place of a condenser and inductance, the mass standing for the inductance and the elasticity for the capacity. The arrangement was very portable. The author's treatment of the conditions of the experiment was a very useful amplification of Vallauri's analysis.

Prof. Howe said he presumed the reason why the time during which the oscillations remained audible could not be extended beyond a few seconds was that harmonics had to be taken into account, whereas the analysis assumed a single sine wave.

Dr. D. Owen said that it was quite possible to hear the note produced in a shunted telephone included in an oscillatory circuit of audible natural frequency, on charging or discharging the condenser. By use of tuning fork and monochord the frequency could be measured, and thus the oscillatory nature of the discharge, and the laws connecting the frequency with inductance and capacity could be verified by the individual student. He had tried to multiply the effect by the aid of a tuning-fork interruptor, but not with much success. The author's method made the demonstration possible to an audience, though at the sacrifice of simplicity. It was to be noted also that the reaction of the mutual inductance would have the effect of slightly raising the frequency of the original circuit. This rise of frequency appeared to be a general consequence of supplying energy for the production of maintained vibrations in an oscillatory system.

XI. *The Quickness of Response of Current to Voltage in a Thermionic Tube.*

By D. Owen, *B.A., D.Sc., and* R. M. Archer, *B.Sc., M.I.E.E., Birkbeck College, London.*

Received November 24th, 1920.

Abstract.

Steady voltages were applied between the hot and cold electrodes of a thermionic tube, for intervals of time which could be varied from 0·00001 second to a minute or longer. The mean current during the interval was measured by the Wheatstone bridge, using a null ballistic method.

Two types of thermionic tube were employed, one at a comparatively high gas pressure, the other at a far higher degree of exhaustion. It is found that on applying the voltage the initial rise of current to its maximum is followed by a fall, the rate of which diminishes with time. In the tube at the high gas-pressure the effect is considerable, and the final value of current may be less than half the initial value. In the case of the tubes at lower gas pressure the fall, though sensible, is far less pronounced, say, three or four per cent.

An experiment is described showing that whatever the change of conditions causing the observed fall may prove to be, this fall is not attributable merely to the high temperature of the filament, but is conditional on the thermionic current being permitted to flow.

The results point to the practical conclusion that in order to avoid phase difference between current and applied voltage, and consequent distortion, at telephonic frequencies, of signals transmitted through a thermionic tube, the vacuum should be as high as possible.

Introductory.

The thermionic tube is now very widely used as an amplifier or relay in telephonic and other circuits. The accuracy of reproduction of the incoming signals is clearly dependent on the absence of any time-lag, or equivalent effect, between the thermionic current flowing in the valve and the voltage impressed on its terminals at the same instant. It is often assumed that owing to the extremely low inertia of the electron, which is the vehicle of the current, the response of current to voltage must be practically instantaneous, and the two therefore necessarily in phase. Thus, for a field of one volt per centimetre the time required for an electron to cover a distance of 1 cm. from rest is less than a thirty-millionth of a second.

There are, however, other factors possibly at work; secondary ionisation of residual gas or of gas liberated by bombardment of the electrodes, or a change in the surface of the hot electrode, may be supposed to be capable of affecting the time required for the attainment of the equilibrium value of the current between grid and filament.

The present Paper contains a description of experiments made to ascertain whether such a time-effect can be placed in evidence. It is shown that in every case tried the answer is in the affirmative. The effect is considerable in tubes containing a platinum (lime-coated) filament, the residual gas pressure being moderately high—well above 0·001 mm. of mercury. In tubes in which the heated filament is of tungsten, and the degree of exhaustion of the bulb far greater, the effect is small, though quite appreciable. In every case the effect observed is a *fall* of current with time; it is thus incorrect to describe it as a time-lag, though it is evident that the outcome is none the less to produce a phase-displacement, and so a distortion of signals in a telephonic or telegraphic circuit in which the thermionic valve is employed.

Experimental Arrangements.

The measurements were carried out by the ballistic method described in a previous Paper.* Consider a Wheatstone bridge connected as in Fig. 1. The grid and anode of a triode valve are put in direct metallic connection, and the space between them and the negative leg of the filament inserted as the unknown resistance in the arm *CD* of the bridge. The battery circuit is closed for a definite period, which can be

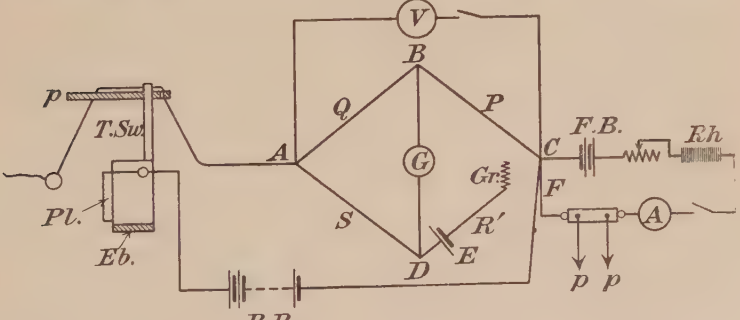

Fig. 1.—Diagram of Electrical Circuits. *T.Sw.*=TIME-SWITCH; *Pl.*=STEEL TARGET; *Eb.*=EBONITE INSULATION; *B.B.*=BRIDGE BATTERY; *F.B.*=BATTERY SUPPLYING FILAMENT; *p p*=LEADS TO POTENTIOMETER; *Gr.*=GRID; *E*=ACCUMULATOR IN SERIES WITH THERMIONIC TUBE.

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* Proc. Phys. Soc., Vol. XXIX., p. 33 (1916).

varied from 0·00001 second to any desired upper limit, the galvanometer being permanently in circuit. The bridge is balanced when the integral flow of electricity through the galvanometer is zero, this condition being attained, as in the ordinary use of the bridge, by variation of the resistance of the adjustable arm.

A difficulty obviously presents itself at the outset when the bridge is applied to the case of a thermionic tube, namely, that even when the battery circuit is open a current will flow from grid to filament, so producing a deflection of the galvanometer. With tubes containing a platinum filament this effect is quite small, and it can be readily shown that observations made with the slightly disturbed zero give sensibly the same results as though the effect were non-existent. With the higher temperatures holding in tubes with tungsten filaments, on the contrary, this effect is quite large. The most convenient way of eliminating it was found to be by inserting an accumulator cell in series in the arm containing the tube; this, of course, involves a change in the subsequent calculation of the resistance at balance, as shown below.

The durations of closure of the battery circuit usually employed were 0·00001, 0·000025, 0·0001, 0·00082, 0·005, 0·05, 1 and 30 sec. The very shortest periods were obtained by the use of balls or rings of steel or brass, which were suspended by fine copper wires. On drawing one of these out of the vertical a definite distance by means of a silk thread, and letting go, collision is made between the ball and a heavy steel target; the battery circuit is thus completed for a period equal to the duration of contact between the ball and the target. These times of contact were ascertained by supplementary measurements involving the use of a standard condenser and a resistance of known and suitable value.

The intermediate times, from 0·005 to 0·02 sec., were obtained by using a pendulum switch. The pendulum, with an attached tongue of hard copper strip, is drawn out a noted distance, then released and allowed to glide over one of two brass strips, one narrow, one wide, let flush into a sheet of ebonite chamfered off to avoid vibratory motion of the tongue. The long closures were made by hand, the balance in these cases being not ballistic, but continuous.

The mean resistance of the space between the cold and hot electrodes, and the current across this space, are calculated (for fuller treatment reference may be made to the Paper already quoted) from the following relations, which hold at balance :—

$$R'=\frac{P}{Q}\cdot S=\frac{v'}{i} \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$v'=V\cdot\frac{P}{P+Q} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$v=v'-E \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$i=\frac{v'}{R'}=\frac{VQ}{S(P+Q)} \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$R=\frac{v}{i}=\frac{(v'-E)(P+Q)S}{VQ} \quad \ldots\ldots\ldots \quad (5)$$

where

P and Q denote the resistances of the ratio arms of the bridge,
S the resistance of variable arm at balance,
R' the apparent resistance of the arm CD (including the thermionic tube in series with the accumulator cell),
R the resistance of the tube alone (hot to cold electrode),
V the voltage applied across the bridge (A to C),
v' the voltage across the arm containing the tube (D to C),
v the voltage across tube only,
E the E.M.F. of accumulator in series in arm containing the tube,
i the current through the tube.

Precautions were taken to ensure the elimination of possible sources of error. It is essential that the voltmeter used in measuring the voltages V should be disconnected during the actual ballistic test, in order to avoid the disturbing effect arising from its inductance. It appeared possible that the inductance of the galvanometer (of the Broca type, resistance 1,000 ohms) might not be without a similar effect. To test this point the connections between B and D were arranged as shown in Fig. 2. A resistance, w, is placed in series with the galvanometer, and in shunt with the two is arranged a $\frac{1}{3}$ mf. mica condenser in series with a resistance, W. The condenser branch exactly compensates the inductive branch if $W=G+w$, and $L/W=KW$, where L denotes the inductance of the galvanometer and G its resistance. With this arrangement the voltage between the points B and D at any instant in the variable state is the same as if these points were connected by a non-inductive resistance W. It was found on making the trial that the balance-point was entirely unaffected by this change, showing that the inductiveness of the galvanometer as simply connected was without sensible effect on the results.

The bridge circuits and the heating circuit were insulated, both from one another and from earth.

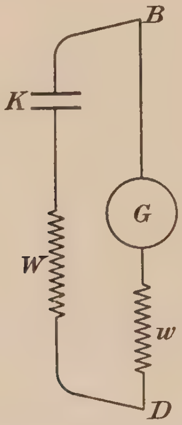

Fig. 2.—Alternative Galvanometer Circuit.

Whenever necessary, extreme constancy of the current through the filament was obtained by the aid of a potentiometer, to which the resistance PS in the heating circuit could be connected.

Experimental Results.

1. *Tube with Platinum (Lime-coated) Filament.*—In this type of tube, as already pointed out, the residual gas pressure is comparatively high. Successive values of voltage from zero to 50 volts were applied between the electrodes of the tube. The observations are represented by the curves of Figs. 3, 4, 5, and 6. Fig. 3 contains the resistance- and current-characteristics over the range 1 to 5 volts, for $t=1$ sec. and $t=0{\cdot}00082$ sec. In Fig. 4 are incorporated the resistance-characteristics over the whole range of voltage used, and for times of application of voltage between the limits of 1 sec. and 0·000025 sec.; the corresponding current-characteristics are given in Fig. 5.

The current falls with time, the effect being very marked except at the higher voltages. Thus at 5 volts the current at the end of 1 sec. is only about two-fifths of its mean value during the first 0·000025 sec. At a voltage of 25 the dependence of current on time has almost disappeared, and this independence persists up to the maximum value of 50 volts. It will be observed that with this tube saturation is far from being attained at that voltage.

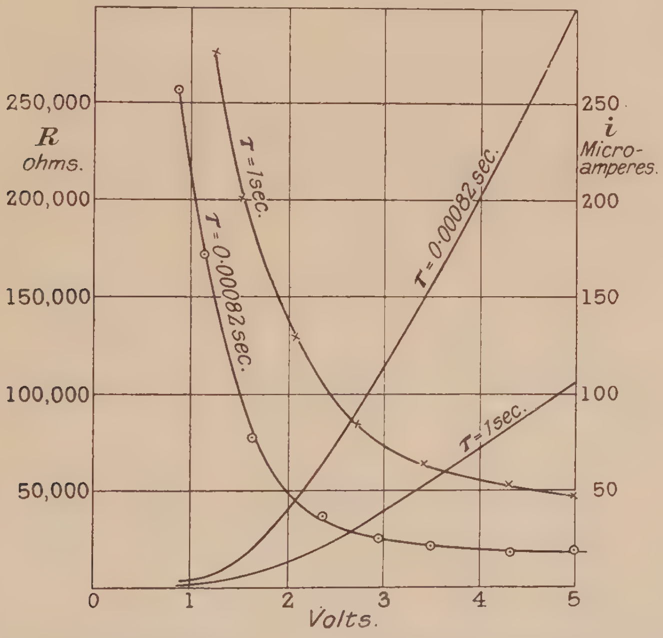

FIG. 3.—RESISTANCE—AND CURRENT-CHARACTERISTICS BELOW 5 VOLTS, FOR $\tau=1$ SEC. AND $\tau=0{\cdot}00082$ SEC. (TUBE WITH PLATINUM FILAMENT. COMPARATIVELY HIGH GAS PRESSURE.)

Within the range employed the current approximates to a limit as the time of application of voltage is continuously reduced. This may be seen by plotting the values of current against time abscissae, or rather, for convenience of scale, against $\log t$, as shown in Fig. 6. This leads to the inference that, within the time-range employed, the current falls with time according to a law of an exponential type, with a time-constant of 0·0065 sec. The maximum and minimum limits of current indicated are 0·9 and 0·25 milli-ampere.

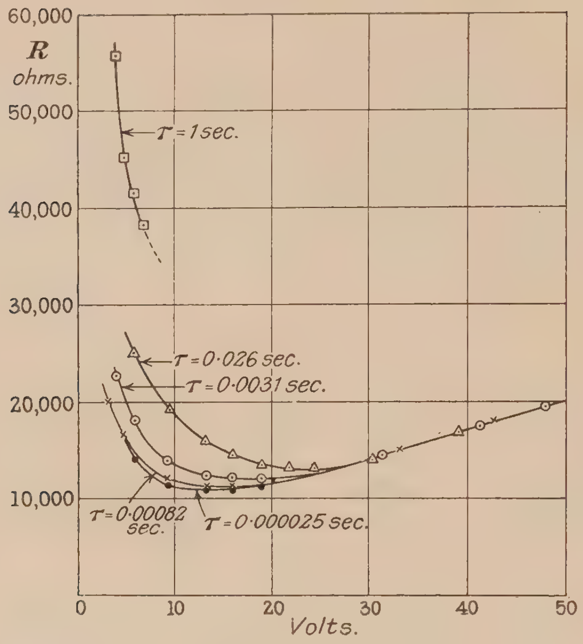

FIG. 4.—RESISTANCE CHARACTERISTICS FOR VARIOUS PERIODS OF APPLICATION OF VOLTAGE FROM 0·000025 SEC. TO 1 SEC., AND FROM 0–50 VOLTS. (TUBE WITH PLATINUM FILAMENT: GAS PRESSURE COMPARATIVELY HIGH.)

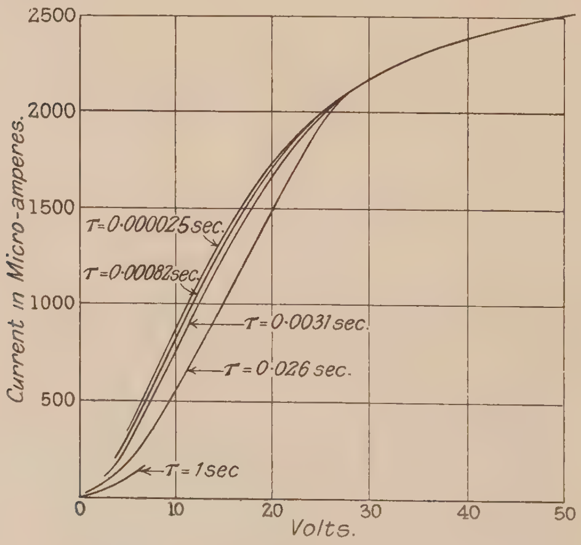

FIG. 5.—CURRENT CHARACTERISTICS. (TUBE WITH PLATINUM FILAMENT.) DERIVED FROM FIG. 4.

2. *Tube with Tungsten Filament.*—These tubes were of the R-type supplied by the General Electric Company. By the courtesy of Mr. C. C. Paterson we are able to state that the residual gas pressure does not exceed one ten-thousandth of a millimetre of mercury, and may be distinctly below that value.

The results may be summed up in a single curve, that of Fig. 7. Three of the characteristics only are shown, for periods of application of voltage equal to 0·0001, 0·005, and 1 sec. respectively. The curve for $t=0·000025$ sec. is indistinguishable from that at 0·000025 sec. Similarly the curve expressing the results for $t=30$ sec. is almost inappreciably different from that at $t=1$ sec.

It is evident that the time-effect is very small in this type of tube, though it unmistakably exists. As before, the effect is a decrease of current with time. The whole variation in this case amounts only to 3 or 4 per cent. A curve of the type of Fig. 6 is also found, a time-constant of the order of 0·0008 sec. being indicated.

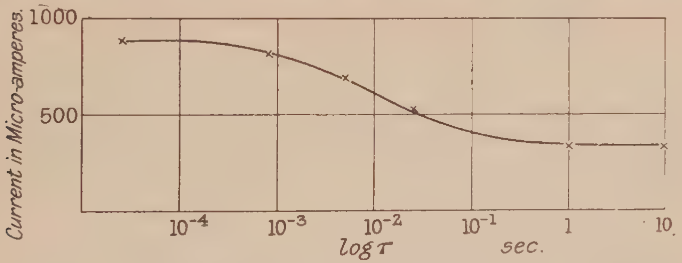

FIG. 6.—THERMIONIC CURRENT PLOTTED AGAINST LOG τ, SHOWING APPROACH TO LIMITING VALUES FOR τ VERY GREAT OR VERY SMALL. (TUBE WITH PLATINUM FILAMENT.)

3. *Experiments on the Influence of the Heating Current.*—It is easy to demonstrate by the experimental method of the present investigation that the cause of the change of condition within the tube which leads to the observed variation of current with time is not to be found in the mere action of the heating current, apart from an actual flow of electrons from it to the cold electrode. Fig. 8 represents the results of a test which extended over 20 hours on a tube with platinum filament. The portion *ABC* gives values of current obtained by ballistic measurement, the period of flow permitted being 0·00082 sec. ; the grid at the intervening times

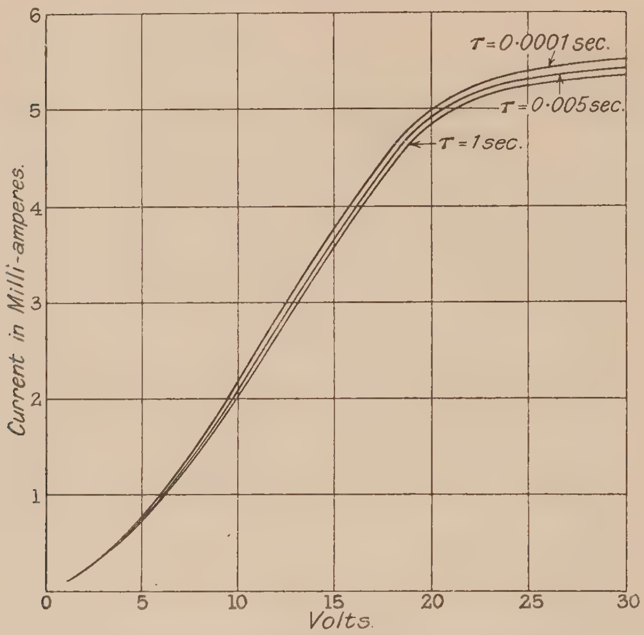

Fig. 7.—Current Characteristic for three periods (τ) of application of voltage. (Tube with tungsten filament. Gas pressure comparatively low.)

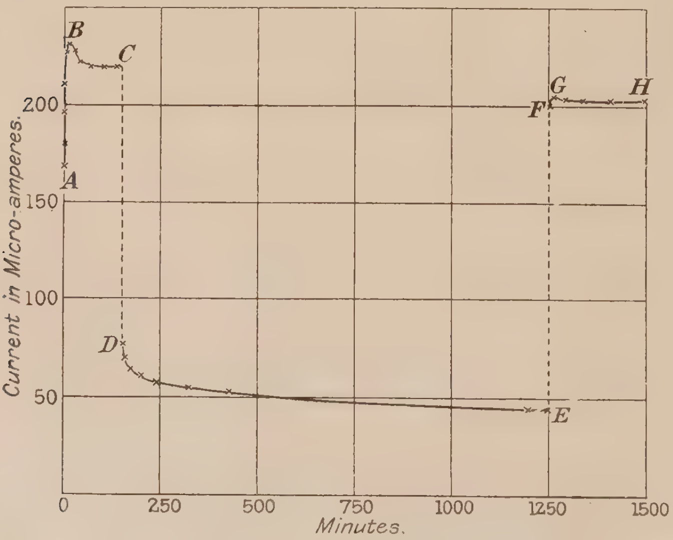

Fig. 8.—Curve showing that the time-variation of current from grid to filament is the result of a change of conditions produced by the flow of this current.

being at —2 volts with respect to the filament, the thermionic flowing was practically zero except during the brief interval of the actual test. The curve indicates the usual course followed by a characteristic during the initial period after switching on the heating current; long before C is reached it has "settled down" to its steady value. On reaching C the voltage (4·39 volts) was permanently applied, and the balance-point obtained about a minute later is represented by the point D. The current has already fallen to less than half its value at C; it continues to fall steadily, and even at E, after 15 hours' steady application of the voltage, the rate of fall is quite appreciable. At E the voltage was removed, and readings again taken every few minutes by short-period applications of the voltage; thus the portion FGH was obtained, showing that the effect of the prolonged flow of thermionic current quickly disappears, though the initial value is not entirely recovered.

Conclusions.

These experiments prove that a sensible time is required to establish the equilibrium value of the current between the hot and cold electrodes in a thermionic tube when a steady voltage is suddenly applied. In every case the current *falls* with time; although the duration of application of voltage was cut down to the limit of 0·00001 sec., in no case was any observation of the initial rise secured.

The cause of the change is assignable to some variation of conditions within the tube, which comes into play only when the thermionic current is allowed to flow. In the type of tube containing a lime-coated platinum filament, for which the residual gas pressure is probably well over 0·001 mm. of mercury, the value of the current after a lapse of some seconds may be as low as one-third of its initial maximum.

In tubes containing a tungsten filament, exhausted to pressures of the order of 0·0001 mm., the total reduction of current amounts only to 3 or 4 per cent.

It is thus probable that the magnitude of the variation of thermionic current with time is determined by the value of the pressure of residual gas in the tube, being continuously reduced as this pressure is reduced.

The distortion of signals of telephonic frequency would probably be quite appreciable in tubes in which the gas pressure is above 0·001 mm., but inappreciable when the pressure is as low as 0·0001 mm.

The earlier work (on the platinum filament type of tube) was carried out in 1915–1916 at the instance of the Department of Wireless Telegraphy, H.M.S. Vernon, Portsmouth.

The authors are indebted to the courtesy of Sir Robert Hadfield for the target of rustless steel, which proved a considerable convenience in the ballistic tests.

DISCUSSION.

Mr. E. V. Appleton pointed out as a possible cause of the effects that the filament is always cooled when the anode current is put on. This means that the electronic emission is reduced. Dr. van der Pol and he had recently had to explain the form of some oscillograms of a triode generator in this way. The condition of the filament before and after closing the anode circuit was quite different, and the effect of the latent heat of electronic evaporation and the heating effect of the anode current had to be taken into account in the latter case. It had been found that because of this the final emission in a certain case was only 86 per cent. of the original amount. Regarding things in this way it is possible to account for the exponential fall of the current with time observed in the present experiments, and if a filament of typical size is considered the time constant comes out about the right value. In soft tubes anything might happen, but he expected that with the filament voltages used a cooling would also be obtained.

Prof. Eccles said he had himself thought of this as a possible explanation. There were also other things which might act in a similar way. If there is a very little gas in the valve a deposit of molecular thickness is formed on the filament and evaporated from it as changes of the anode current occur. This layer greatly alters the electron emission. Another point which had occurred to him was that the change in the distribution of the electron cloud which must occur when the voltage is applied might be responsible, but a consideration of the times involved ruled this out. Besides resistance these tubes had a species of back E.M.F., and, as far as possible, attempts should be made to separate them in dealing with valve problems.

Dr. Rayner said that if the effects were due to cooling of the filament the dimensions of the latter ought to have an important influence.

Mr. F. E. Smith pointed out that the straight part of the curve *DE* (Fig. 8) was approximately parallel to *CG*. This seemed to indicate that the steady drop between 250 and 1,250 minutes had nothing to do with the anode voltage being on, but was probably due to fatigue of the filament.

A Visitor queried the authors' statement regarding Fig. 2. The inductance of the path *BGD* was unaffected by the shunt circuit *BKD*.

Dr. Vincent suggested that it was desirable to indicate by a ring, in Fig. 1, the existence of glass round the valve to show that it was a vacuum tube. His first impression on reading the Paper was that our usual ideas as to characteristics would have to be scrapped. He was relieved to find that the curves of Fig. 7, for the hard valve, were so nearly the orthodox characteristics taken in the usual way.

Dr. G. D. West emphasised the importance of the study of the internal action of valves, and congratulated the authors on their work.

Dr. Owen, in reply to the discussion, said : The theories of the variation of thermionic current with time which have been put forward by speakers, are of great interest. The authors in the present Paper had made it their concern to establish the existence and the magnitude of this effect, in the case of the two classes of valve—namely, the soft and the hard types. They would be able to pursue their further work in the light of the explanations advanced, and perhaps to discriminate between them. With respect to the query as to the arrangement of Fig. 2, the important point was that the *combined* circuit behaved like a perfectly non-inductive resistance connecting the points B and D, thus avoiding the persistence of an induced E.M.F. acting on the part of the bridge outside BD. How this was brought about, and the state of affairs in BGD and BKD, constituted an interesting though simple problem, which the questioner would be able to solve after a little examination. In reply to Dr. Vincent, he hoped that the fact that valves were enclosed in glass tubes was sufficiently known to render unnecessary the expense incurred in altering the block.

Mr. Archer made some remarks explanatory of the devices used in the production of the short intervals of time used in the experimental work described in the Paper.

On the Mechanical Equilibrium of a Sphere of Gravitating Fluid. By CHARLES H. LEES, *D.Sc., F.R.S.* (*Part of the Presidential Address to the Physical Society, February* 13, 1920.)

(*Continued from Vol. XXXII., p.* 270.)

9. IN many cases it is more convenient to define the state of the homogeneous medium at radius r in terms of the pressure p and temperature θ, rather than the pressure and specific volume v of the fluid. We then write

$$\frac{dv}{dr}=\left(\frac{\partial v}{\partial p}\right)_\theta \frac{dp}{dr}+\left(\frac{\partial v}{\partial \theta}\right)_p \frac{d\theta}{dr},$$

or, since in virtue of the pressure equation

$$\frac{dp}{dr}=-\frac{g}{v},$$

where g is the gravitational acceleration at radius r,

$$\frac{dv}{dr}=-\left(\frac{\partial v}{\partial p}\right)_\theta \frac{g}{v}+\left(\frac{\partial v}{\partial \theta}\right)_p \frac{d\theta}{dr}.$$

The condition for stable mechanical equilibrium (6·2) then becomes

$$-\left(\frac{\partial v}{\partial p}\right)_\theta \cdot \frac{g}{v}+\left(\frac{\partial v}{\partial \theta}\right)_p \cdot \frac{d\theta}{dr} > g\beta_\phi,$$

or

$$\left(\frac{\partial v}{\partial \theta}\right)_p \cdot \frac{d\theta}{dr} > g(\beta_\phi-\beta_\theta) \quad . \quad . \quad . \quad . \quad . \quad (9·1)$$

where β_θ is the isothermal compressibility of the medium.

Thus, for stable equilibrium

$$\frac{d\theta}{dr} > -g\frac{\beta_\theta-\beta_\phi}{\left(\frac{\partial v}{\partial \theta}\right)_p}.$$

But

$$\beta_\theta-\beta_\phi=\frac{1}{c_p} \cdot \frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p^2. \quad . \quad . \quad . \quad . \quad (9·2)$$

Hence the condition for stable equilibrium becomes

$$\frac{d\theta}{dr} > -\frac{g}{c_p}\,\frac{\theta}{v} \cdot \left(\frac{\partial v}{\partial \theta}\right)_p,$$

i.e.,

$$-\frac{d\theta}{dr} < \frac{g}{c_p} \cdot \frac{\theta}{v} \cdot \left(\frac{\partial v}{\partial \theta}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9·3)$$

where c_p is the specific heat of the medium at constant pressure expressed in work units.

10. If we apply this condition to a homogeneous ideal gas well away from the centre of the fluid sphere we have for the gas

$$pv=\frac{R}{m}\theta.$$

Hence $$\frac{\theta}{v}\left(\frac{\partial v}{\partial\theta}\right)_p=1,$$

and the condition for stability becomes

$$-\frac{d\theta}{dr}<\frac{g}{c_p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10{\cdot}1)$$

Or, since for the gas

$$c_p=\frac{R}{m}\cdot\frac{\gamma}{\gamma-1},$$

we may put the condition in Kelvin's form

$$-\frac{d\theta}{dr}<\frac{\gamma-1}{\gamma}\,\frac{g}{R}\cdot m. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10{\cdot}2)$$

As the specific heats and their ratio are constant for such a gas, we see that while g is constant there will be no tendency for one portion of the gas to become unstable for the same temperature slope before another. The decrease of g as we proceed outwards through the outer and rarer portion of the gas will, however, reduce the margin of stability for a given temperature slope.

For a gas or vapour well away from the centre, to which Van der Waal's equation applies, we have

$$\left(p+\frac{a}{v^2}\right)(v-b)=\frac{R}{m}\theta,$$

where R is nearly the same in value for all gases, and

$$\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial\theta}\right)_p=1+\frac{1}{v}\left(\frac{2am}{R\theta}-b\right). \quad . \quad . \quad . \quad . \quad . \quad (10{\cdot}3)$$

For gases for which the Joule-Kelvin effect is a cooling

$$\frac{2am}{R\theta}-b$$

is positive. For hydrogen at temperatures above 193° absolute it is negative, but becomes positive at lower temperatures. For most gases c_p increases as the temperature rises. Hence the variations of $\frac{\theta}{v}\left(\frac{\partial v}{\partial\theta}\right)_p$ and of c_p may oppose each other in their effect on the stability. The effect of gravity is the same as in the case of a perfect gas.

For a homogeneous liquid not near the centre $\frac{1}{v}\left(\frac{\partial v}{\partial\theta}\right)_p$ is of the order 0·001 and increases about 10 per cent. per 100°C. c_p is of the order $1{\cdot}5\times 10^7$, and increases at a somewhat greater rate. The variation of stability will, therefore, depend mainly on the product $g\theta$, the factors of which may change in the same or in opposite directions as we move radially. Where $g\theta$ is least the stability condition is most exacting.

11. If the fluid is not homogeneous throughout but consists of a number of homogeneous media which do not mutually dissolve each other, the various media will for stable equilibrium be arranged in concentric spherical shells, subject to the equivalent conditions that in each homogeneous fluid shell

$$\frac{dv}{dr} > g\beta_\phi, \quad \frac{d\varphi}{dr} + ve \quad \text{and} \quad -\frac{d\theta}{dr} < \frac{g}{c_p}\cdot\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial\theta}\right)_p,$$

while at each spherical surface separating two media the specific volume v must not decrease as r increases.

On each side of the separating surface $\frac{d\varphi}{dr}$ must be positive, while at the surface itself φ changes from φ_1 for the first to φ_2 for the second medium. We may, therefore, state the law of change of entropy with r in the form · —. For stable equilibrium $\frac{d\varphi}{dr}$ at any point must exceed the rate of increase of φ due to change of composition of the fluid at that point.

The fact that ice is in equilibrium on the surface of water shows that the entropy per gram of the upper material need not always exceed that of the lower.

As the conditions for thermal equilibrium at a surface separating two fluids are that the heat fluxes and temperature on the two sides of the surface shall be equal, it is sometimes convenient to write the condition for stable mechanical equilibrium in each fluid in the form

$$-k\,\frac{d\theta}{dr} < k\frac{g}{c_p}\cdot\frac{\theta}{v}\left(\frac{\partial v}{\partial\theta}\right)_p \quad . \quad . \quad . \quad . \quad . \quad . \quad (11{\cdot}1)$$

where k is the rate of transmission of heat per sq. cm. per second, for unit slope of temperature.

Thus the flux of heat outwards per square centimetre per second must be less than

$$k\frac{g}{c_p}\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial\theta}\right)_p,$$

that is than $k\frac{g}{c_p}\cdot\theta\cdot a_p$ where $a_p=\frac{1}{v}\left(\frac{\partial v}{\partial\theta}\right)_p$, and is sometimes called the coefficient of cubical dilatation.

12. Although it will not often happen that the equilibrium just becomes stable in each of two contiguous fluids at the same instant, it may be mentioned that in such an event there would, in general, be an abrupt change in the value of $\frac{dv}{dr}$ on passing through the surface of contact. If v_1 is the specific volume and β_1 the adiabatic compressibility of the first and $v_2\beta_2$ of the second fluid, we have on one side of the surface of contact $\frac{dv_1}{dr}=g\beta_1$ and on the other $\frac{dv_2}{dr}=g\beta_2$. In the same way the value of $-\frac{d\theta}{dr}$ changes from $\frac{g}{c_1}\cdot\frac{\theta}{v_1}\left(\frac{\partial v_1}{\partial\theta}\right)_p$ to $\frac{g}{c_2}\frac{\theta}{v_2}\left(\frac{\partial v_2}{\partial\theta}\right)_p$ and of φ from φ_1 to φ_2.

Thus in passing upwards from a liquid to a gas or vapour in both of which equilibrium is attained at the same instant, since the adiabatic compressibility of the gas or vapour is much greater than that of a liquid, the rate of increase of the specific volume with height will be much greater in the gas or vapour than in the liquid. Since $\frac{1}{v}\left(\frac{\partial v}{\partial\theta}\right)_p$ for a gas is of the order $3{\cdot}6\times10^{-3}$ and for a liquid of the order 10^{-3}, while the specific heats of gases and vapours are, in general, less than those of liquids, the value of $-\frac{d\theta}{dr}$ in the gas or vapour will be of the order 5 times that in the liquid just below it, if the two attain equilibrium together.

As the condition for thermal equilibrium requires the values of $-\frac{d\theta}{dr}$ to be inversely as the heat transmission rates of the gas and liquid, which are about in the ratio 1 : : 10, as a rule the condition for mechanical equilibrium commencing in the two together will not also give thermal equilibrium.

In the case of dry air at the surface of the earth convective equilibrium exists when $-\frac{d\theta}{dr}$ is 1°C. in 100 metres, in sea water at temperatures between 0°C. and 30°C., when $-\frac{d\theta}{dr}$ is $\frac{1}{200}$°C. in 100 metres. On the other hand, the rate of heat transmission in sea water is about 30 times that of air. Hence "convective" and thermal equilibrium could not occur together in air and sea water.

13. Although the probability of two contiguous layers of fluid attaining the "convective" or neutral equilibrium state together is small, they may subsequently be in stable mechanical and thermal equilibrium with each other. If k_1 is the thermal transmission in the first fluid, the condition for stable mechanical equilibrium in that fluid may be written $-k_1\frac{d\theta_1}{dr} < k_1\frac{g}{c_1}\frac{\theta_1}{v_1}\left(\frac{\partial v_1}{\partial \theta_1}\right)_p$. Similarly, in the second the condition is $-k_2\frac{d\theta_2}{dr} < k_2\frac{g}{c_2}\frac{\theta_2}{v_2}\cdot\left(\frac{\partial v_2}{\partial \theta_2}\right)_p$. The conditions for thermal equilibrium are $\theta_1 = \theta_2$, $-k_1\frac{d\theta_1}{dr} = -k_2\frac{d\theta_2}{dr}$ at the interface.

Both mechanical and thermal conditions are satisfied if at the interface the equal quantities $-k_1\frac{d\theta_1}{dr}$ and $-k_2\frac{d\theta_2}{dr}$ are less than the smaller of the two quantities $k\frac{g}{c_p}\cdot\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p$ for the two fluids respectively. For air this has a value considerably less than for water. Hence the condition for stability in the air is last to be satisfied.

14. Although convection currents in the sphere while in the unstable state will, in general, ensure the complete mixture of miscible fluids, there may be cases in which two miscible fluids are not completely mixed before the equilibrium becomes stable. In order to examine this point we assume that the fluid is not homogeneous but consists of a mixture in varying proportions of a number of fluids which mutually dissolve each other. We see from §5 that stable equilibrium will only be possible if the specific volume of the mixture is

uniform throughout each thin concentric shell and increases from shell to shell as we proceed outwards.

The equilibrium here referred to is a molar equilibrium, and its existence does not preclude diffusion of the various fluids into each other, which is a molecular, not a molar, motion. The diffusion is superposed on the phenomena we are discussing.

The condition for stable equilibrium is still given by the inequality

$$\frac{dv}{dr} > g\beta_{\phi},$$

in which $\frac{dv}{dr}$ is the complete rate of increase of the specific volume of the mixture with radius and β_{ϕ} is the adiabatic compressibility of the mixture at radius r, but the corresponding conditions involving the entropy and temperature are slightly modified.

15. In the case of the entropy condition we write the compressibility equation in the form

$$\frac{dv}{dr} > -\frac{g}{v}\left(\frac{\partial v}{\partial p}\right)_{\phi . M},$$

where the subscript M indicates that the mass of each constituent in 1 gram of the mixture is constant, and by means of the relation

$$\left(\frac{\partial v}{\partial p}\right)_{\phi . M} = -\left(\frac{\partial \varphi}{\partial p}\right)_{vM} \Big/ \left(\frac{\partial \varphi}{\partial v}\right)_{pM},$$

convert it into

$$\left(\frac{\partial \varphi}{\partial v}\right)_{pM} \frac{dv}{dr} > \frac{g}{v}\left(\frac{\partial \varphi}{\partial p}\right)_{vM}. \quad . \quad . \quad . \quad . \quad . \quad (15{\cdot}1)$$

Now

$$\frac{d\varphi}{dr} = \left(\frac{\partial \varphi}{\partial v}\right)_{pM} \frac{dv}{dr} + \left(\frac{\partial \varphi}{\partial p}\right)_{vM} \frac{dp}{dr} + \sum_{n}\left(\frac{\partial \varphi}{\partial M_n}\right)_{vp} \frac{dM_n}{dr}, \quad . \quad (15{\cdot}2)$$

where M_n is the mass of the nth constituent and $\Sigma M_n = 1$.

On substituting in this equation the condition for stability, and $\frac{dp}{dr} = -\frac{g}{v}$, we get as the entropy condition for stability

$$\frac{d\varphi}{dr} > \sum_{n}\left(\frac{\partial \varphi}{\partial M_n}\right)_{pv} \frac{dM_n}{dr}. \quad . \quad . \quad . \quad . \quad . \quad (15{\cdot}3)$$

That is the rate of increase of the entropy with radius due to *all* causes must exceed that which would be due to change of composition of the fluid mixture if the pressure and specific volume remained constant.

16. To find for the mixture the modification of the condition of stability expressed in terms of the temperature slope outwards, we again write the general compressibility condition in the form

$$\frac{dv}{dr} > -\frac{g}{v} \cdot \left(\frac{\partial v}{\partial p}\right)_{\phi \,.\, M}.$$

Taking p, θ and the mass M_n of each of the n constituents of the mixture where $\Sigma M_n = 1$, as the variables defining the mixture at r we get

$$\left(\frac{\partial v}{\partial p}\right)_{\theta \,.\, M} \frac{dp}{dr} + \left(\frac{\partial v}{\partial \theta}\right)_{pM} \cdot \frac{d\theta}{dr} + \sum_n \left(\frac{\partial v}{\partial M_n}\right)_{p\theta} \frac{dM_n}{dr} > -\frac{g}{v}\left(\frac{\partial v}{\partial p}\right)_{\phi \,.\, M}.$$

But

$$\frac{dh}{dr} = -\frac{g}{v}.$$

Hence the condition for stability becomes

$$\left(\frac{\partial v}{\partial \theta}\right)_{pM} \cdot \frac{d\theta}{dr} + \sum_n \left(\frac{\partial v}{\partial M_n}\right)_{p\theta} \cdot \frac{dM_n}{dr} > \frac{g}{v}\left\{\left(\frac{\partial v}{\partial p}\right)_{\theta M} - \left(\frac{\partial v}{\partial p}\right)_{\phi M}\right\},$$

that is using (9·2)

$$\frac{d\theta}{dr} > -\frac{g}{c_p} \cdot \frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_{pM} - \frac{\sum_n \left(\frac{\partial v}{\partial M_n}\right)_{p\theta} \cdot \frac{dM_n}{dr}}{\left(\frac{\partial v}{\partial \theta}\right)_{pM}},$$

or

$$-\frac{d\theta}{dr} < \frac{g}{c_p} \cdot \frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_{pM} + \frac{\sum_n \left(\frac{\partial v}{\partial M_n}\right)_{p\theta} \cdot \frac{dM_n}{dr}}{\left(\frac{\partial v}{\partial \theta}\right)_{pM}}$$

$$< \frac{g}{c_p} \cdot \frac{\theta}{v} \cdot \left(\frac{\partial v}{\partial \theta}\right)_{pM} + \frac{\frac{1}{v}\sum_n \left(\frac{\partial v}{\partial M_n}\right)_{p\theta} \cdot \frac{dM_n}{dr}}{\frac{1}{v}\left(\frac{\partial v}{\partial \theta}\right)_{pM}}.$$

The last term is the quotient of the fractional increase of specific volume with radius which would be due to change of composition at constant pressure and temperature, by the coefficient of cubical dilatation at constant pressure, and represents the rate of change of temperature with radius which at constant pressure would give the same change of specific volume as the actual change of composition of the fluid with the radius would at constant pressure and temperature.

The process of diffusion diminishes the value of this term, and therefore diminishes the stability of the fluid for a given temperature slope. If the diminution is sufficient, the stability may cease, and convection currents begin to facilitate the mixture of the constituents.

17. Although the three forms of the condition for stability for a sphere of gravitating fluid dealt with in the foregoing pages are equivalent, it appears desirable to retain all three. The form $\frac{dv}{dr} > g\beta_\phi$ holds in all cases, the second—that the rate of increase with the radius of the entropy per gram must exceed that due to change of composition only, and the third—that the temperature slope outwards or the flux of heat outwards per square centimetre per second must be less than a certain minimum, are connecting links between the problem of mechanical stability here considered and that of thermal stability, which is of equal importance.

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CONTENTS.

PAGE

V. Some Slide Rule Improvements. By J. St. Vincent Pletts 57

VI. The Current Density in the Crater of the Carbon Arc. By N. A. Allen, B.Sc. 62

VII. A Sodium Vapour Electric Discharge Tube. By F. H. Newman, M.Sc., A.R.C.Sc., University College, Exeter 70

VIII. Absorption of Gases in the Electric Discharge Tube. By F. H. Newman, M.Sc., A.R.C.Sc., University College, Exeter 73

IX. Magnetic Separation of Neon Lines and Runge's Rule. By H. Nagaoka, Professor of Physics, Imperial University of Tokyo 83

X. A Method of Demonstrating the Retroactive Property of a Triode Oscillator. By E. V. Appleton, M.A., M.Sc. 100

XI. The Quickness of Response of Current to Voltage in a Thermionic Tube. By D. Owen, B.A., D.Sc., and R. M. Archer, B.Sc., M.I.E.E., Birkbeck College, London...... 104

On the Mechanical Equilibrium of a Sphere of Gravitating Fluid. By Charles H. Lees, D.Sc., F.R.S. (Part of the Presidential Address to the Physical Society, Feb. 13, 1920). (*Continued from Vol. XXXII., p. 270.*) 116